Particle Filtering by a Planetary Gap. Wm. R. Ward, Southwest Research Institute, Boulder, CO 80302.

It has been recently suggested that gaps created in a disk by an embedded planet, M_p, could be very efficient particle filters [1,2]. Drag forces from the super(sub)-keplerian rotation of gas caused by the density gradient at the outer(inner) wall of the gap would cause an outward(inward) drift of particles, resisting their penetration of the gap. The effect is size dependent, with larger particles drifting faster. This mechanism could inhibit solids enrichment of giant planets [1].

Lets briefly review how the filtration mechanism works. The gas pressure gradient causes the azimuthal velocity of the gas to differ from Keplerian by an amount $\Delta v_g = -\eta r\Omega$, where $\eta = -h^2(d\ln\sigma_g/d\ln r)/2$, with $h = c/r\Omega$ being the normalized disk scale height, c the gas sound speed, r, Ω the orbital distance and frequency of the planet, and σ_g the gas surface density. Particles do not respond to the gas gradient directly, but gas drag alters their azimuthal velocity so that it too differs from Keplerian by $\Delta v_p = -\eta r\Omega/(1+\tau_s^2)$, where $\tau_s = \rho_s d/\sigma_g$ is the non-dimensional stopping time for particles of diameter d and density ρ_s due to gas drag. This drag also changes the particles' angular momentum, which causes a radial drift

$$u_p = -2\eta r\Omega\tau_s/(1+\tau_s^2) \approx h^2\tau_s r\Omega(d\ln\sigma_g/d\ln r) \quad (1)$$

where we have assumed $\tau_s \ll 1$. Unopposed, this would cause particles to drift out of the gap [1]. Further consideration [2] as to whether a radial gas velocity $u_g = F/2\pi r\,\sigma_g$ due to a global inward disk flux, $F \approx -10^{-9}M_\odot/yr$, could carry particles into the gap concluded that particles larger than $\sim 10\mu m$ would be filtered out of the gas.

There are two issues that are not addressed in previous studies that we wish to explore here:

(*i*) If, as typically assumed, disk evolution is due to turbulence, the fluctuating velocity field of the gas generates a velocity dispersion in the particles as well, which will acquire a particle diffusivity of order ν/Sc , where ν is the turbulent viscosity of the gas and Sc is the Schmidt number that is of order unity for $\tau_s \ll 1$. This dispersion might give rise to diffusive transport of particles into the gap in spite of a systematic outward drift promoted by gas drag.

(*ii*) Particle loading affects the gas dynamics and can alter the predicted particle radial drift rate [3]. Once the particle density exceeds that of the gas, the drift rate drops quickly making it more difficult to hold off particle diffusion into the gap. A two fluid model [3] can be employed to show that the gas and particle azimuthal velocity perturbations become

$$\Delta v_g = -\left[1 - \frac{\sigma_p/\sigma_T}{1+\tau_s^2(\sigma_g/\sigma_T)^2}\right]\eta r\Omega \qquad (2a)$$

$$\Delta v_p = -\frac{\sigma_g/\sigma_T}{1+\tau_s^2(\sigma_g/\sigma_T)^2}\eta r\Omega \qquad (2b)$$

where σ_p is the particle surface density, $\sigma_T = \sigma_g + \sigma_p$, and we have assumed the particles and gas are well mixed vertically. Again, the azimuthal drag force exchanges angular momentum between gas and particles inducing the latter to drift radially at the rate

$$u_p = -2\eta r\Omega\tau_s(\sigma_g/\sigma_T)^2/(1+\tau_s(\sigma_g/\sigma_T)^2)$$

$$\approx (\sigma_g/\sigma_T)^2 h^2\tau_s r\Omega(d\ln\sigma_g/d\ln r) \qquad (3)$$

Eqn. (3) has an additional factor $(\sigma_g/\sigma_T)^2$ compared to eqn. (1). This is because when $\sigma_p > \sigma_g$, particles begin to control the gas and force its motion to be more closely Keplerian, which weakens the drag force. This suggests that as more particles are delivered to the gap edge by the general nebula flow, the drag induced outward drift may become unable to prevent diffusion of particles into the gap. As the particle annulus extends further and further into the gap the particle density will be higher at the planet. Indeed, if the density at the planet rises to the point where the particle/gas density ratio equals that of the ambient disk, a quasi-steady state could be achieved wherein the rate of particle accretion is the same as the global delivery rate.

To test this hypothesis, we employ a two-component model treating gas and the particles as separate fluids coupled by gas drag. For simplicity, we will seek a gap stationary state with zero gas and particle fluxes. The usual approach is to balance the gradient of the viscous couple, $g = 3\pi\sigma\nu r^2\Omega$, with the tidal torque density from the planet, $dT_p/dr \sim \varepsilon\mu^2 r^2\Omega^2\sigma r/x^4$,where $\varepsilon = +(-)$ for the outer(inner) gap wall, $\mu = M_p/M_\odot$, and x is the distance to the planet normalized to its orbital radius [4]. Here we do this for each component, but include an additional torque density, dT_{dg}/dr, due to their mutual drag, *i.e.*,

$$3\pi\nu r^2\Omega\frac{\partial\sigma_g}{\partial r} = \epsilon\mu^2 r^2\Omega^2\sigma_g\frac{r}{x^4} - \frac{dT_{dg}}{dr} \quad (4)$$

$$3\pi(\nu/Sc)r^2\Omega\frac{\partial\sigma_p}{\partial r} = \epsilon\mu^2 r^2\Omega^2\sigma_p\frac{r}{x^4} + \frac{dT_{dg}}{dr} \quad (5)$$

where we have ignored slow changes in g. Setting Sc ~ 1 and adding these leads to the standard equation for the total surface density, indicating the same profile, *i.e.*,

$$\sigma_T = \Sigma e^{-|W/x|^3}, \quad W \equiv (\mu^2 r^2\Omega/9\pi\nu)^{1/3} \quad (6)$$

as a one-component (gas) disk, where Σ is the ambient, unperturbed disk surface density. Since the tidal torque does not itself change the component fractions, we eliminate it by multiplying eqns. (4) and (5) by σ_p and σ_g respectively, and then differencing them to find

$$3\pi\nu r^2\Omega(\sigma_p\frac{\partial\sigma_g}{\partial r} - \sigma_g\frac{\partial\sigma_p}{\partial r}) = -\sigma_T\frac{dT_{dg}}{dr} \quad (7)$$

The gas torque comes from the tangential gas drag and we deduce its strength from the particle drift rate eqn. (3), *i.e.*, $dT_{dg}/dr = \pi\sigma_p r^2\Omega u_p$. We find

$$dT_{dg}/dr = \pi(\sigma_p\sigma_g/\sigma_T^2)\rho_s drc^2(d\ln\sigma_g/d\ln r) \quad (8)$$

Next we introduce the particle-to-gas ratio $\gamma = \sigma_p / \sigma_g$ and a viscosity $\nu = \alpha c^2/\Omega$. Substituting (8) into (7) and eliminating σ_p in favor of γ, yields after some rearranging,

$$\sigma_g^2 d\gamma/d\sigma_g = (\rho_s d/3\alpha)\gamma/(1+\gamma) \quad (9)$$

Integration of (9) and writing the gas density in terms of the total density, $\sigma_g = \sigma_T/(1+\gamma)$, gives

$$\frac{\sigma_T}{\Sigma} = \frac{1+\gamma}{1+\gamma_o+\Lambda[\gamma_o-\gamma+\ln(\gamma_o/\gamma)]} \quad (10)$$

which is a two parameter family of curves, where γ_o is the solids-to-gas ratio just beyond the gap and $\Lambda \equiv 3\alpha\Sigma/\rho_s d$. Eqn. (10) gives the disk surface density at which given solid/gas ratio γ is reached. This could be combined with eqn. (6) to find the corresponding distance from the planet. Variations of the individual components' surface densities are easily found as well.

Figure 1a shows eqn. (10) for the case of $\alpha = 10^{-3}$, $\Sigma = 100 g/cm^2$, $\gamma_o = 0.01$ for various particle sizes d. The dotted line shows the density at the edge, $w_H = 2.47(\mu/3)^{1/3}r$, of the horseshoe zone for a Jovian sized planet where accretion can occur. We find that particles smaller than ~0.1mm are able to penetrate the gap to w_H, although larger particles would tend to accumulate near the gas edge. However, these calculation have ignored the global inward flux of the disk [2]. If this is considered, one would expect γ_o at the gap edge to increase with time. Figure 1b shows the penetration of 1mm sized particles as a function of γ_o up to 100 (which is comparable to that needed for gravitational instability of the particle layer). A particle enrichment to $\gamma_o \lesssim O(1)$ is sufficient to raise $\gamma(w_H)$ to the ambient disk value of $O(10^{-2})$. Even larger particles could penetrate at still higher γ_o, α, or Σ. Larger particles will also tend to settle into a thinner layer, increasing the mid-plane solid-to-gas ratio. This suggests that a gap may not be able to filter out particles indefinitely.

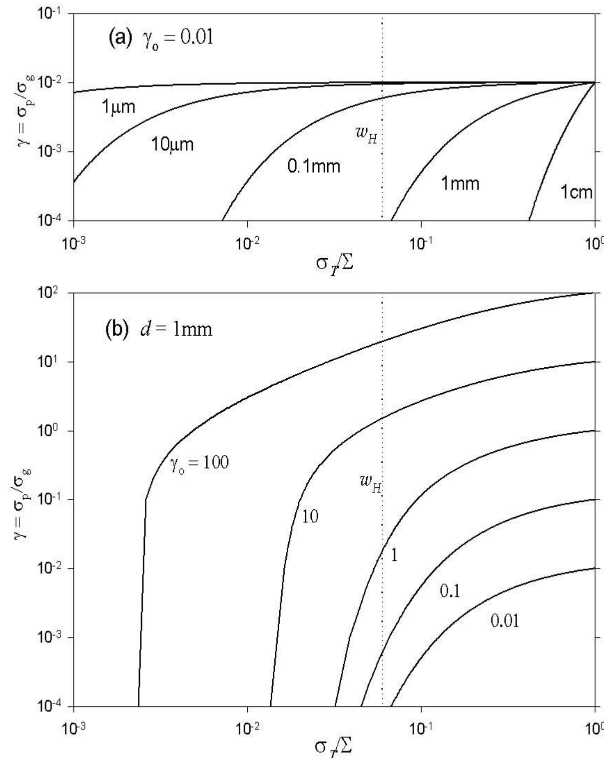

Figure 1. (a) γ as a function of the gap surface density for $\gamma_o = 10^{-2}$ and various particle diameters d, and (b) for 1mm particles with various assumed γ_o.

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