HYPERBOLIC ORBITS AND THE PLANETARY FLYBY ANOMALY. H.-J. Blome[1] and T. L. Wilson[2], [1]University of Applied Sciences, Hohenstaufenallee 6, 52066 Aachen, Germany, [2]NASA, Johnson Space Center, Houston, Texas 77058 USA.

Introduction: Space probes in the Solar System have experienced unexpected changes in velocity known as the flyby anomaly [1], as well as shifts in acceleration referred to as the Pioneer anomaly [2-4]. In the case of Earth flybys, ESA's Rosetta spacecraft experienced the flyby effect and NASA's Galileo and NEAR satellites did the same, although MESSENGER did not – possibly due to a latitudinal property of gravity assists.

Measurements indicate that both anomalies exist, and explanations have varied from the unconventional to suggestions that new physics in the form of dark matter might be the cause of both [5]. Although dark matter has been studied for over 30 years, there is as yet no strong experimental evidence supporting it [6]. The existence of dark matter will certainly have a significant impact upon ideas regarding the origin of the Solar System. Hence, the subject is very relevant to planetary science.

We will point out here that one of the fundamental problems in science, including planetary physics, is consistency. Using the well-known virial theorem in astrophysics, it will be shown that present-day concepts of orbital mechanics and cosmology are not consistent – for reasons having to do with the flyby anomaly. Therefore, the basic solution regarding the anomalies should begin with addressing the inconsistencies first before introducing new physics.

Virial Theorem and Hyperbolic Trajectories: Earth fly-by's and the Pioneer anomaly both involve close orbital encounters at small impact parameters resulting in hyperbolic trajectories with respect to a planet in the Solar System. They both involve nonlocal transitions from bound to unbound states in one fashion or another, derived from patched-conic techniques used in astrodynamics. We will now show that energy cannot be conserved in such a procedure by virtue of the virial theorem. The necessity for resolving this issue necessarily raises an old theme in cosmology, the question of whether or not cosmological expansion has an effect upon the dynamics of local systems [7-9, 4].

The virial theorem provides a general relation between the time-averaged total kinetic energy $T=<T>$ and potential energy $U=<U>$ such that the virial energy $2T+U$ is zero: $2T+U=0$. It applies for a self-gravitating system of equal-mass objects (stars, galaxies, etc.) in stable equilibrium and has been used to examine the stability of galactic clusters believed to have negative total energy E, the classical definition of a bound state. Briefly, a stable bound-state system's potential energy must equal its kinetic energy within a factor of two.

The Cosmic Virial Theorem. The Layzer-Irvine equation [10-11] is an extension of the virial theorem to systems that interact with an expanding cosmic environment ($\dot{R}/R > 0$). It relates the total system energy $E = T+U$ with the virial energy $2T+U$ as follows:

$$\frac{d}{dt}(T+U) + \frac{\dot{R}}{R}(2T+U) = 0 \quad , \qquad (1)$$

where $\dot{R}$ is the expansion parameter and $H = \dot{R}/R$ is the Hubble parameter. Note that (1) is similar to the cosmological perturbations of local systems found by Cooperstock et al. [8] and is consistent with the results of Anderson [7].

Expansion causes the energy of a system *not* in virial equilibrium to change because from (1)

$$-\frac{d}{dt}(T+U) = \frac{\dot{R}}{R}(2T+U) = 0 \qquad (2)$$

which can happen *only* when the virial energy is zero, $2T+U = 0$. Otherwise total energy E cannot be conserved in accordance with the left-hand side of (2).

Space Astrodynamics. For a general Keplerian orbit, (2) reads:

$$-\frac{d}{dt}(-\frac{GMm}{2a}) = \frac{\dot{R}}{R}\left\{m(V_r^2+V_\phi^2) - \frac{GMm}{r}\right\} \qquad (3)$$

or

$$-\frac{d}{dt}(-\frac{GM}{2a}) = \frac{\dot{R}}{R}\left\{\frac{GMe(1+e\cos\phi)}{a(1-e^2)}\right\} \qquad (4)$$

for velocity V, semi-major axis a, eccentricity e, and true anomaly φ.

Bound-State Orbit (Circular, e=0). For a bound-state Keplerian orbit with zero eccentricity, (4) becomes

$$-\frac{d}{dt}(-\frac{GM}{2a}) = \frac{\dot{R}}{R}\{0\} = 0 \quad , \qquad (5)$$

and it follows from the left-hand side of (2) that total energy E is conserved.

Unbound Orbit (Hyperbolic Trajectory, e>1). For an unbound Keplerian orbit on a hyperbolic trajectory, (4) becomes

$$-\frac{d}{dt}(-\frac{GM}{2|a_H|}) = -\frac{\dot{R}}{R}\left\{\frac{GMe(1+e\cos\phi)}{|a_H|(e^2-1)}\right\} \neq 0 \qquad (6)$$

where a_H is the semi-major axis for the hyperbolic case and $a_H < 0$. It is obvious that the total energy $E=T+U$ is not conserved in (6) due to the left-hand side of (2).

This means that hyperbolic trajectories in (6) such as that for Pioneer are influenced by cosmic expansion while closed conical orbits like those in (5) are not. The source of the nonconservation of energy is the cosmic perturbation of local dynamics. The Layzer-Irvine equation, then, confirms the argument [7] that cosmic expansion couples to escape orbits while it does not couple to bound orbits. This would readily "explain" the Pioneer acceleration anomaly as due the expanding Universe.

Inconsistencies: It makes common sense that the total energy of the Universe *E* should be conserved, if physics is to be self-consistent. However, demonstrating this is another matter. Many things in physics are unobservable and surely this is one of them.

At the root of the problem is the metric assumed in General Relativity (GR). One adopts a metric such as the Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology [12] for an expanding Universe. For Solar System dynamics, one adopts the Schwarzschild metric for the classical solutions predicted by GR. These are hardly the same thing, save for being coupled to one another by the miniscule cosmological constant. One can embed the latter metric into the former and patch the interface(s) with boundary conditions between the two as did Einstein & Straus [13-14], but the procedure is *ad hoc* and contrived. Such hybrid metrics are often referred to as "swiss cheese" models. In truth, no one has ever succeeded in dealing with this problem using GR. At best, one can study it using perturbation methods such as post-Newtonian approximations.

Interpretations: The very purpose of the virial theorem in astrophysics and cosmology is to address the physical behavior of a stable, self-gravitating, spherical distribution of equal-mass objects (stars, galaxies, globular clusters, etc.). The subject is not entirely metaphysics (beyond physics) and falls within the purview of general astrophysics.

There is another way to paraphrase the results (1)-(6) above. There is a discrepancy between the virial theorem that has been used to investigate the short-term stability of globular clusters [15] and the gravitational dynamics of local systems. This has been pointed out by Bonnor as well [16] who used the Einstein-Straus results to show that if the field equations of GR apply aptly well to small-scale systems (such as the Solar System), they may not apply at all to the large-scale system of an expanding Universe if the energy-momentum tensor is constructed by averaging over all of the small-scale constituent systems.

There appears to be a discrepancy or inconsistency in the way the energy-momentum tensor in relativity is defined for small-scale systems and the virial energy is defined for stable large-scale systems in astrophysics and cosmology – else they are unstable. At issue is the stability of massive systems and whether their total energy is negative (bound) or positive (unbound). In particle physics this is never a discrepancy because of unitarity that is introduced in the scattering matrix. The global scattering problem in gravitation apparently is an entirely different matter.

Conclusions: By virtue of the virial theorem, the nonconservation of energy for a flyby-type observer while changing from bound-state elliptical trajectories to hyperbolic ones has been pointed out. This result illustrates how the physics of such trajectory techniques for interplanetary flybys and gravity assists using patched-conics is still not understood at a very fundamental level.

References: [1] Anderson J. D. et al. (2008) *Phys. Rev. Lett. 100*, 091102. [2] Anderson J. D. et al. (2002) *Phys. Rev. D 65*, 082004. [3] Anderson J. D. et al. (1998) *Phys. Rev. Lett. 81*, 2858-2861. [4] Wilson T. L. and Blome H.-J. (2009) *Adv. Spa. Res.*, to appear. [5] Adler S. (2009) *Phys. Rev. D*, to appear. [6] Amsler C. et al. (2008) *Phys. Lett. B 667*, 1-1340. [7] Anderson J. L. (1995) *Phys. Rev. Lett. 75*, 3602-3604. [8] Cooperstock et al. (1998) *Ap. J. 503*, 61-66. [9] Adkins G. S. et al. (2007) *Phys. Rev. D 75*, 064011. [10] Layzer D. A. (1963) *Ap. J., 138*, 174-184. [11] Irvine D. (1961) *PhD Thesis*, Harvard University. [12] Blome H.-J., Wilson T. L. (2005) *Adv. Space Res. 35*, 111-115. [13] Einstein A. & Straus E. (1945) *Rev. Mod. Phys. 18*, 120-124. [14] Einstein A. & Straus E. (1946) *Rev. Mod. Phys. 17*, 148-149. [15] Burbridge E. M. and Burbridge G. R. (1961) *Astron. J. 66*, 541-550. [16] Bonnor W. B. (1987) *Ap. J. 316*, 49-51.