DETERMINISTIC NEUTRON TRANSPORT MODELING FOR PLANETARY APPLICATIONS. R. Furfaro[1], P. Panfili[2], A. Luciani[2], J. S. Kargel[3], B. Ganapol[1,3], A. Palmero-Rodriguez[4], D. Mostacci[2]. [1]Aerospace and Mechanical Engineering Department, University of Arizona (robertof@email.arizona.edu), [2]Dipartimento di Ingegneria Energetica, Nucleare e Controllo Ambientale, Università di Bologna, [3]Hydrology and Water Resources, University of Arizona, [4]Planetary Science Institute.

Introduction: Neutron spectroscopy has been successfully used to estimate the distribution of hydrogen on planetary surfaces [1,2]. For example, the Gamma Ray Spectrometer (GRS) suit mounted on Mars Odyssey collected neutron and gamma ray signals indicating large amount of water ice at a Martian latitude of 60 degrees and above [2], as confirmed by recent in-situ measurements performed by the Phoenix Lander. On the Moon, measurements obtained by Lunar Prospector were used to obtain maps of the major compositional units including ability to discriminate between basaltic and feldspatic units and determining the abundance of subsurface water ice [1]. The mathematical relationship between geophysical properties of the observed surface and measured neutron fluxes is generally determined by using Montecarlo-based techniques [3] which are able to simulate complex geometries and multi-particle transport phenomena. However, such techniques are computationally expensive and only few data points can be realistically simulated. Therefore, full-scale inversion methods cannot be applied and geophysical properties can be estimated with higher degree of uncertainty.

Here, we present the development of a deterministic approach to simulate the energy-dependent transport of neutrons in physical settings typical of planetary surfaces. Starting from first principles, we apply the conservation of photons to derive the multi-group, multi-layer, steady-state transport equation that computes the depth-depedent flux distribution of neutrons both below the surface and leaking out of the surface.

Neutron Flux Modeling via Deterministic Transport Equations: The basic principle behind particle transport in planetary media is conservation of neutrons. If the planetary magnetic field is absent (e.g. Mars, Moon, Mercury), Galactic Cosmic Rays (GCR) penetrate the planet's surface and interact with the nuclei of the elements comprinsing the surface. As result of nuclear reactions, neutrons are generated and transported thoughout the medium as some are redistributed within the subsurface and others leak out in the outer space. At steady-state neutrons must be balanced in the appropriate phase-space, i.e. neutron production is counter-balanced by the net flux of neutrons streaming out. The linearized Boltzmann equation describes mathematically the balance. However, assumptions are generally made to render the equation mathematically tractable. For planetary media we assume that 1) the neutron transport occurs in the vertical direction (one spatial variable), 2) the transport is azimuthally averaged (one-angle approximation), 3) the neutron energy is constrained to belong within a finite number of level (multi-group approximation), 4) the planetary surface is assumed to be comprised of a set of homogeneous layers (multi-layer approximation). Considering the simple case of a surface comprised of one single homogeneous layer, the Boltzmann equation becomes:

$$\mu \frac{\partial}{\partial \tau} \mathbf{\Psi}(\tau,\mu) + \mathbf{\Sigma}\mathbf{\Psi}(\tau,\mu) = \qquad (1)$$
$$= \frac{1}{2}\sum_{l=0}^{L} P_l(\mu)\mathbf{C}_l \int_{-1}^{1} P_l(\mu')\mathbf{\Psi}(\tau,\mu')d\mu' + \mathbf{Q}(\tau,\mu)$$

for $\tau \in (0,\tau_0)$ and $\mu \in [-1,1]$. Here the Legendre polynomials are denoted by $P_l(\mu)$, $\Sigma_i = s_i / s_{\min}$ define the elements of the diagonal $\mathbf{\Sigma}$ matrix where $\{s_i\}$ are the total cross sections for each group and $s_{\min}$ is the minimum of the total cross-section set; the transfer matrices are defined by $\mathbf{C}_l = \mathbf{T}_l / s_{\min}$ where matrices $\mathbf{T}_l$ are defined by the group transfer cross sections. To be complete, we also note that z is the spatial variable, $\tau = z s_{\min}$ and $\tau_0 = z_0 s_{\min}$, μ is the direction cosine that defines the direction of propagation, the flux vector $\mathbf{\Psi}(\tau,\mu)$ has the angular fluxes for each energy group $\Psi_i(\tau,\mu)$ for i = 1, 2, … , G, as the defined components and the term $\mathbf{Q}(\tau,\mu)$ represents the neutron source we are considering inside the medium.

Eq (1), equipped with appropriate boundary conditions can be extended to the multi-layer case. The Analitical Discrete Ordinate (ADO) method as proposed by Siewert [4] is then adapted to provide fast and accurate solutions. After discretizing the angle in 2N direction, Eq(1) is reduced to a set of ordinary differential equations that can be solved by finding the homogeneous and inhomogeneous solutions (eigenvalue problem). The homogeneous solution is expressed as function of undetermined constants that must be determined by imposing boundary conditions. While zero flux is assumed entering at the outer boundary of the top and bottom layer, flux continuity is imposed at the interface between materials. The inhomogeneous solution can be found applying the

Green's function formalism [4]. The numerical procedure has been implemented in MATLAB and has been tested against benchmarks available in the literature [5]. Further comparison with more conventional SN techniques have been performed to establish numerical consistency [6].

Model Validation on Mars Odyssey data: Following Boynton et. al. [2], a three-layer transport model has been devised to simulate thermal and epithermal neutron fluxes leaking out of the Martian surface. Two goals have been identified, i.e. 1) show that the deterministic model gives results comparable to montecarlo simulations and 2) show that there is good agreement between the measured and simulated data. The three-layer model consists in a top atmospheric layer, a dry intermediate layer (1.5 % in water weight) and an ice-rich (variable percentage) bottom layer. Soil has been assumed to have the same composition as determined by the Pathfinder X-Ray Spectrometer. Atmosphere is assumed to have an average of 13 g/cm^2 [8]. We assume the source of fast neutrons to have an exponential decay and we use the intensity as well as the spatial decay parameter to calibrate our transport model. Figure 1 shows a comparison between the measured and simulated thermal/epithermal fluxes. The figure shows good agreement with the same calculation performed by Boynton et. al. [2] using a montecarlo approach. Figure 2 shows a direct comparison of measured and simulated thermal/epithermal fluxes as function of the latitude. The model has been calibrated by changing the fast neutron source parameters. The simulations indicates a content of water in the upper layer which can vary between 1.5 and 2% and a lower layer whose depth decreases and water ice content increases moving toward the pole. In particular, poleward of -60°, the water rich layer contains 60 +/- 10% water by weight and is covered by less than 20 +/- 5 g/cm^2 of dry materials. This is consistent with data reported in the literature [8].

Conclusions and Future Work: A fast and accurate deterministic transport model for computing neutron fluxes from planetary surfaces has been described. The greatest advantage of the described approach against previously employed probabilistic codes (i.e. Montecarlo) is its simplicity and speed. The model has been validated on Mars data and its performances on Lunar Prospector data are being evaluated. Inverse model-based techniques (.e.g. neural networks) for fast and accurate surface and subsurface water detection are under development [7].

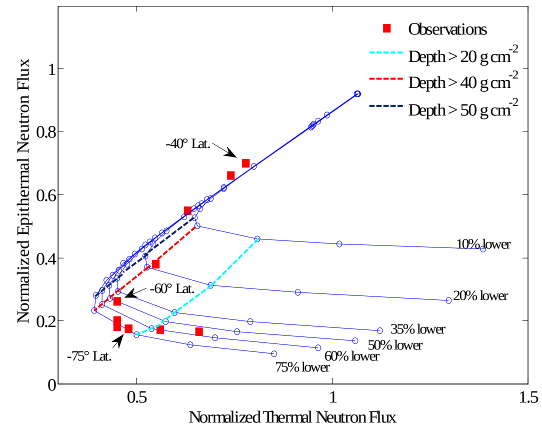

Figure 1: Calculated and observed normalized epithermal neutron flux versus normalized thermal neutron flux for two layered models in which the upper layer has 1.5% H_2O, the lower layer H_2O content is indicated in the figure, and the depth of the lower layer varies as indicated in the text. Observed fluxes at different latitudes are plotted as unconnected squares.

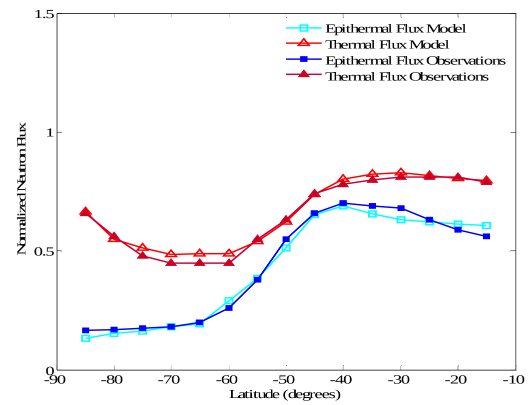

Figure 2: Comparison between simulated and observed normalized neutron fluxes versus latitude.

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