A SHAPE MODEL FOR RHEA AND IMPLICATIONS FOR ITS GRAVITY COEFFICIENTS AND INTERNAL STRUCTURE. S. W. Asmar[1], F. Nimmo[2], P.C. Thomas[3], B.G. Bills[1]. [1]Jet Propulsion Laboratory, California Institute of Technology, Pasadena CA 91109, asmar@jpl.nasa.gov, [2]Department Earth and Planetary Sciences, University of California Santa Cruz, 1156 High St, Santa Cruz, CA 95064, USA, [3]Center for Radiophysics and Space Research, Cornell University, Ithaca, NY 14853.

Introduction: Models of the interior structure of planets and their satellites rely heavily on the determination of their gravitational potential. In addition, the shapes of the bodies reveal important information about subsurface processes, such as relaxation to a gravitational equilibrium and, when combined with mean density, a quantification of departure from a homogeneous structure [1]. We have determined the shape of the Saturnian icy satellite Rhea in the form of a spherical harmonic expansion to degree 10 from limb profile data obtained from Cassini imaging [1]. We also investigated whether the gravitational effects of this surface topography are sufficient to have biased existing estimates of the degree-2 gravity field.

Radio tracking of range and range-rate measurements to Cassini gave estimates of the mass and quadrupole gravitational moments of Rhea [2,3]. Estimates of the zonal gravitation harmonic J_2 and sectoral harmonic C_{22} have led to varying conclusions, from an undifferentiated interior (assuming the hydrostatic constraint $3J_2 = 10C_{22}$) [4], to models favoring an "almost undifferentiated" satellite [2], to the latest analysis [3] exploring a broader range of geophysical assumptions as well as the implications of the presence of degree-3 and 4 gravity coefficients, and concluding that Rhea is non-hydrostatic.

Only if Rhea is hydrostatic can measurements of its quadrupole gravitational moments be used simply to determine its internal structure. Whether Rhea is hydrostatic or not can be tested by measuring J_2 and C_{22} and seeing whether their ratio has the hydrostatic ratio of 10:3. Unfortunately, estimation of J_2 and C_{22} may be complicated by the presence of unestimated higher-order gravitational coefficients [3]. One source of such higher-order terms is uncompensated surface topography, which is our interest here.

Method: We determined the shape of Rhea using topography from 22 limb profiles [1]. The topography was expanded into spherical harmonic coefficients up to *l=m=10* via a least-squares fit method with the appropriate normalization of the Legendre functions. We verified the ability of the code to recover the correct coefficients using synthetic data placed at the locations of the real profiles.

Results: Table 1 lists the low-order spherical harmonic topography coefficients. Figure 1 shows the coordinates of the limb profiles on a map of Rhea [1]. Figure 2 shows the topography derived from spherical harmonic expansion superimposed on a map of Rhea. Higher-order (short wavelength) features are less reliably constrained than lower-order features. There is little evident correlation with topographic features observed in the images, such as the large impact basin at ~30°N, 150°W.

The amplitude of the degree-3 topographic terms is ~0.3 km (Table 1), or ~$4\text{x}10^{-4}$ in dimensionless units. If this topography is uncompensated, it will generate gravity anomalies which may be aliased into the degree-2 gravity coefficients determined during flybys [3]. Applying equation (5) of [5] and assuming that the topography represents uncompensated ice (density 0.9 g/cc), the dimensionless degree-3 gravitational coefficients will be $\sim 100\times10^{-6}$. The predicted degree-3 gravity potential coefficients assuming uncompensated surface topography are therefore about an order of magnitude smaller than the measured degree-2 coefficient ($J_2=931\times10^{-6}$) in [3].

If this topography really is uncompensated, will it affect the determination of the degree-2 gravity coefficients? It was argued in [3] that an unestimated degree-3 gravity coefficient exceeding $28\text{x}10^{-6}$ (normalized) will degrade the fit to the quadruople-only (degree-2) gravity coefficients. The degree-3 gravity coefficients we predict are a factor of ~4 larger than this critical value. It is therefore likely that the quadrupole-only solution will have been contaminated by these higher-degree terms. In particular, if the topography is uncompensated, then the size of the degree-3 terms suggests strongly that a hydrostatic Rhea cannot currently be excluded.

The above conclusion is apparently paradoxical: if uncompensated topography is invoked, how can Rhea be hydrostatic? In fact, this is not an unreasonable scenario: Rhea's degree-2 shape and gravity field were probably established early on, while it was warm and hydrostatic. Subsequently, as the shell cooled and developed elastic strength, later events (mainly impacts) will have generated shorter-wavelength uncompensated topography and corresponding gravity anomalies.

Future Work: Given the above discussion, it is obviously important to test whether the surface topography is truly uncompensated. We will do so by examining how the Doppler residuals are altered when higher order gravity terms due to uncompensated or partially-compensated surface topography are included. A reduction in the residuals would suggest that the ice is indeed incompletely compensated, but would leave uncertain the level of compensation of the rock underneath.

The topographic variance spectrum derived from our spherical harmonic coefficients is notably different from the $\sim l^{-2}$ dependence observed for silicate bodies [6]. Why this should be the case, and whether the spectral shape contains any information about the degree of compensation, is the subject of ongoing investigations.

Conclusions: We have derived a low-order shape model for Rhea, which has an amplitude of several km at these wavelengths. If this surface topography is uncompensated, it will generate gravity anomalies large enough to bias existing estimates of the degree-2 gravity coefficients. The predicted anomalies mean that it is currently not possible to rule out a hydrostatic Rhea. This conclusion, while negative, is important: if Rhea is hydrostatic, then the possibility of determining its internal structure [2,4] remains open. If, however, Rhea turns out to be significantly non-hydrostatic at degree-2, then determining its internal structure from gravity coefficients will prove much more challenging. Alternative approaches, such as using its obliquity to determine a moment of inertia [7], may be more fruitful.

References: [1] Thomas P. C. et al., *Icarus* **190**, 573-584, 2007. [2] Iess et al., *Icarus* **190**, 585-593, 2007. [3] Mackenzie et al., *GRL* **35** L05204, 2008. [4] Anderson and Schubert, *GRL* **34** L02202, 2007. [5] McKenzie and Nimmo, *Icarus* **130**, 198-216, 1997. [6] Kaula, *GRL* **20**, 2583-2586, 1993. [7] Bills and Nimmo, *Icarus* **196**, 293-297, 2008.

l	***m***	C_{lm}	S_{lm}
2	0	-0.14406	
2	1	-0.30758	-0.01784
2	2	0.08116	0.36158
3	0	0.03298	
3	1	-0.35626	-0.19287
3	2	-0.06989	0.03412
3	3	0.30688	0.39561

Table 1: Low-order spherical harmonic coefficients for Rhea topography (in km).

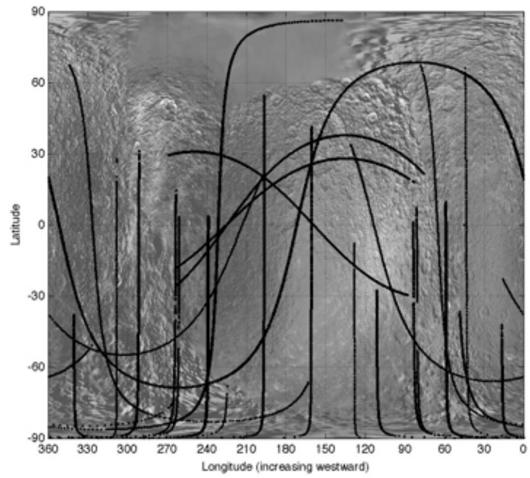

Figure 1: A projection of a map of Rhea with the limb profiles [1] used for the shape model.

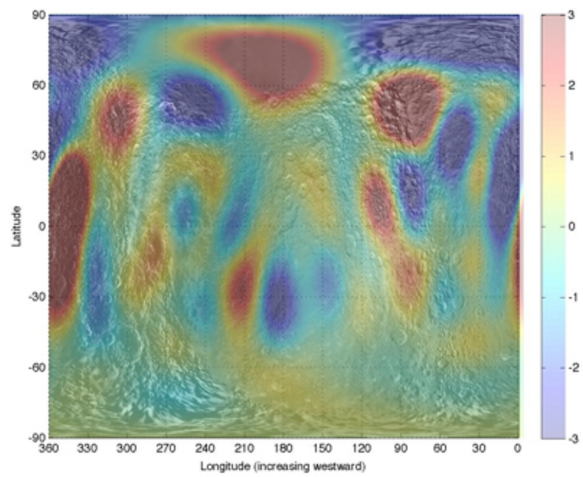

Figure 2: A map of the topography derived from spherical harmonic expansion superimposed on a map of Rhea (color scale in km).