A NEW SOLUTION OF THE LUNAR GRAVITY FIELD USING LOCALIZED SPECTRAL CONSTRAINT.

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Introduction: The lunar gravity has long been a challenge in planetary geodesy. Until the recent Japanese SELENE mission, and its 4-way Doppler data, no direct radio tracking data over the far side of the Moon were available [1]. In 2011, the NASA GRAIL mission will provide uniform high-quality inter-satellite tracking and greatly expand our knowledge of lunar gravity [2].

In preparations for the Lunar Reconnaissance Orbiter (LRO) mission [3], the historical tracking data of lunar orbiters were reprocessed with the GEODYN II orbit determination program [4] using the latest force models and planetary ephemerides. The data sets include Lunar Orbiter 4 and 5, the Apollo 15 and 16 subsatellites, Clementine, and Lunar Prospector (LP). New solutions were obtained to degree 150, using regular inversion schemes such as a global Kaula constraint [5]. Here, we present parallel efforts to improve the initial gravity field to be used during the Precision Orbit Determination of LRO.

Principle: Planetary gravity fields have typically been inverted, described and used in terms of spherical harmonics. When the least-squares information matrix is ill-defined, the solution needs to be regularized. That is, the power of all the coefficients of the spherical harmonics expansion at degree l is fixed *a priori*. This 'Kaula rule' is empirical, different for each planetary body, but was found to be approximately satisfied to high degree when unconstrained solutions became possible. Because of the "far side gap" in tracking data, lunar gravity field solutions have to be constrained even at relatively low degrees ($l>20$) [6]. Due to the global spatial support of the spherical harmonics functions, this means that the near side gravity field, despite excellent coverage and data quality, can be affected by the errors of the solution on the far side.

Here, we use the technique described in [7] to perform the least-squares solution with localized spherical harmonics (with global-support, satisfying the Laplace equation). The goal is to prevent the problems associated with the lack of coverage of the far side from leaking into the near side. Each of those functions, the so-called Slepian functions described in [8], is a linear combination of (global) spherical harmonics functions. Computationally, this is significant, as it means we can use the normal equations created for the regular spherical harmonics solution, instead of reprocessing the whole data and creating an alternate set of normal equations. Indeed, unlike [7], which used the line-of-sight residuals of the Lunar Prospector Doppler data to regionally improve the gravity field to high degrees, we invert the full normal equations. The set of normal equations used in the global solution can be combined linearly to become a new set of equations for the localized solution.

The associated localized spherical harmonics basis functions will for the most part have their power concentrated either nearly fully inside or outside the chosen circular cap region. The region size and the cut-off concentration ratio can be optimized with respect to the data coverage. Here, we preliminary chose a cap centered on (0°,0°) with an half-angle of 100° as the primary region (near side), and a concentrating factor of 0.5. The historical tracking data coverage is sufficient for the inversion of the gravity field to high degree over the near side, but the ill-determination of the far side made the use of a constraint necessary. Thus, over the far side, we use the same Kaula rule as in the global case, but in the near side region, no constraint is applied to let the data fully determine the solution.

Results: Given that the majority of the information on the near side lunar gravity field is due to the Lunar Prospector mission, the preliminary solution presented here does not include other data. In the near future, the normal equations created with the data of other spacecraft will be included. Lunar Prospector is especially important at high resolution because of its extended mission at a mean altitude of ~50km, sometimes much lower. However, the Apollo 15 and 16 data are very valuable for the inversion of the far side gravity field. Their low altitude and low inclination orbits, combined with the low altitude and polar orbits of Lunar Prospector help localize the larger gravity anomalies on the far side. But in globally constrained solutions, the Apollo 15 and 16 ground tracks tend to be visible, even on the near side, as the large far side residuals 'leak'. To avoid this problem, we will in future work evaluate how different weighting of those data on the near and far sides can allow their maximum use for the far side while not affecting the near side, already well determined by Lunar Prospector.

Figure 1 illustrates the difference between a globally constrained solution and the localized constraint solution. The changes are small over the far side region, which is expected given that there both the constraint and the data are identical. Major differences are visible near the limbs. The boundary between the two regions is clearly visible, and shows that in this preliminary solution, the lack of any constraint near the limbs is too optimistic. Although LP observes the spacecraft as it flies near the limbs, the geometry is not optimal and the orbit is nearly face-on. Given the data density of Lunar Prospector data there, some Kaula-type constraint needs to be applied, at least at high degrees around the limbs. This could be done either by reducing the size of the near side region, or by tapering the constraint weight between both regions.

On the other hand, many of the features observed outside of the limb regions on the near side appear to be better correlated with topography. Figure 2 shows that the localized constraint solution has more power at high degrees than either the JPL LP150Q solution [6] or the global constraint LP-only solution. It also shows that the correlation with topography is improved over the near side compared to either solution when that region is left unconstrained.

The polar correlation with topography is improved, and simulations performed with an artificial gravity field as 'truth' show similar changes as in the polar regions of Figure 1. The numbered circular regions of Figure 1, which show higher than average differences with the global constraint solution, suggest that these are improvements to the field (e.g., Figure 3).

Summary: In the case of the Moon, where the lack of tracking data over the far side is a problem for the usual inversion techniques based on global spherical harmonics, the use of localized constraint to regularize the solution where needed is a promising way to use the full potential of the available tracking data sets while keeping the far side stable at high degrees. Current efforts focus on the shortcomings of the results presented here. We expect that new solutions will still show their benefits, while the instability in the limb regions are reduced.

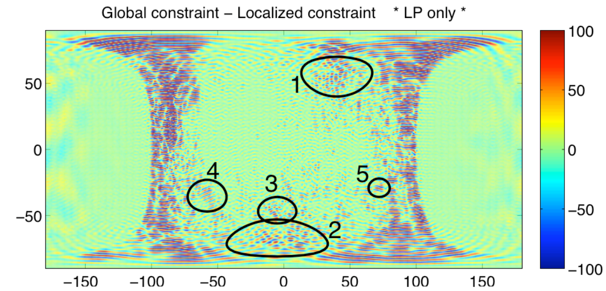

Figure 1. Difference of the global and localized solutions, in mgals.

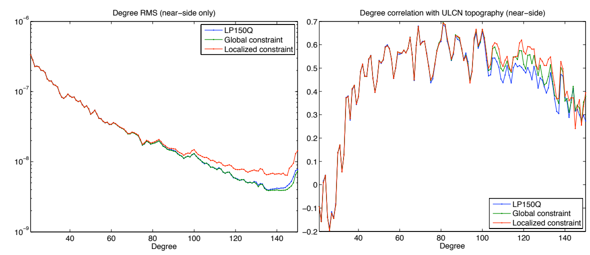

Figure 2. [left] Near side power spectra of LP150Q [6] (blue), global (green) and localized (red) constraint solutions. **[right]** Near side degree correlation of the same gravity fields with the ULCN2005 topography.

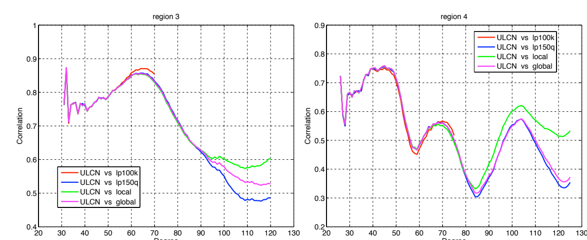

Figure 3. Localized correlation within regions 2 (left) and 4 (right), shown on Figure 1, of the LP100K (a priori, red), LP150Q (blue), global (magenta) and localized (green) constraint gravity solutions, with the ULCN2005 topography

References:
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