Statistical Tests of Turbulent Concentration of Chondrules.

Seth A. Teitler[1], Julie M. Paque[2], Jeffrey N. Cuzzi[3], and Robert C. Hogan[4]. [1]Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL, USA (setht@uchicago.edu), [2]California Institute of Technology, Division of Geological and Planetary Sciences, Pasadena, CA, USA, [3] NASA Ames Research Center, Moffett Field, CA, USA, [4]BAER, inc., Mountain View, CA, USA

Introduction. Chondrules from different chondrites show evidence of having undergone some process of sorting by size and density [1]. One hypothesized mechanism for sorting chondrules is turbulent concentration (TC) [2]. The hypothesis involves a sorting process that acts only on the product of radius $r$and density ρ for each chondrule, normalized by a factor that depends on conditions in the nebula at the site of concentration. The normalization factor would thus be constant over all chondrules in a given chondrite, but might vary from one chondrite to the next. Using radius and density data for eight sets of chondrules from seven different chondrites [2,3] and simulated samples generated from numerical simulations of TC [2], we conduct several statistical tests. The hypotheses we test are: (1) Are the chondrite samples consistent with lognormal or Weibull size distributions as previously suggested [4]; (2) Is there evidence in favor of aerodynamic sorting (determined by ρr) over alternative combinations of ρ and r; and finally (3) Are the chondrite samples consistent with the specific aerodynamic sorting mechanism of TC? Our meteorite data sample has been expanded beyond the sample of six chondrites explored qualitatively in [2] and [3], using unpublished data taken by one of us (JMP) on two additional chondrule sets: chondrules picked from QUE 93030, and chondrules picked from GRO 95524. In all cases, individual chondrules were either sonically disaggregated from cohesive chondrite samples using freeze-thaw cycling, or picked from loose samples, and the radius and density of each chondrule were measured ([3]). The statistical tests have been carefully selected to handle various obstacles in the formulation of the hypothesis and the nature of the data sets being compared. The eight samples were compared pairwise (see below), giving us a total of 28 pairwise tests (21 tests when excluding the OTTA80301 data set; see below).

Test Methods. One obstacle to simple statistical analysis of the TC hypothesis is the assumption that the data from each chondrite must be normalized by an unknown factor that varies between chondrites. That is, the mean chondrule size varies from one chondrite group to the next, influenced by variable (and unknown) nebula gas density or turbulent intensity ([2]). As a result, the tests used must allow for shifting the location parameters (typically taken to be the empirical means) of the data sets so they match. A second obstacle is that the concentrations given by numerical simulations of TC are themselves essentially a sampled, discrete set of (numerically derived) values, which must be interpolated to yield a predicted concentration curve. Tests of the predicted concentration curve must then be distribution-free, since the true curve is not a known analytic function.

Kernel density estimates of a chondrite data set using a Gaussian kernel (a generalization of a histogram that assigns a Gaussian function to each data point and sums to give an estimate of the parent distribution curve) are compared with the best-fit lognormal for the data set and the interpolated TC prediction curve in Figure 1. The kernel density estimates, while continuous, reflect the discreteness and statistical fluctuations in the original chondrule data and simulations. The comparison suggests that the chondrite data set is inconsistent with its best-fit lognormal and is consistent with being drawn from the TC distribution function. This conclusion is supported by the quantitative tests described below.

The most convenient statistical tests given the assumptions of the turbulent concentration mechanism are tests which compare individual samples in a pairwise fashion. These pairwise tests were used to test all the hypotheses below. In order to test the simultaneous hypothesis for all of the chondrite data sets, we employ the Benjamini-Hochberg procedure [5,6] for multiple comparisons. All tests were run at a 95% significance level for the appropriate simultaneous hypothesis using the Benjamini-Hochberg procedure.

Test 1: To test the hypothesis that the chondrules are drawn from a sorting curve of lognormal or Weibull form in ρr or r, we used the Anderson-Darling test [7]. This is an extension of the more common Kolmogorov-Smirnov (K-S) test, but unlike K-S it can handle the requirement of shifting the means in each pairing of data sets without losing power and significance.

Test 2: To test the general hypothesis that chondrules are sorted by some aerodynamic mechanism (which would imply sorting by ρr, with a normalization factor for each chondrite, but does not imply sorting according to the specific concentration curve predicted by TC), we used Miller's jackknife test [8,9] on pairs of chondrite data sets, yielding 28 pairwise (observation vs. observation) comparisons. Miller's jackknife test is an asymptotically distribution-free test of differences in dispersion for data sets which have unknown and possibly unequal

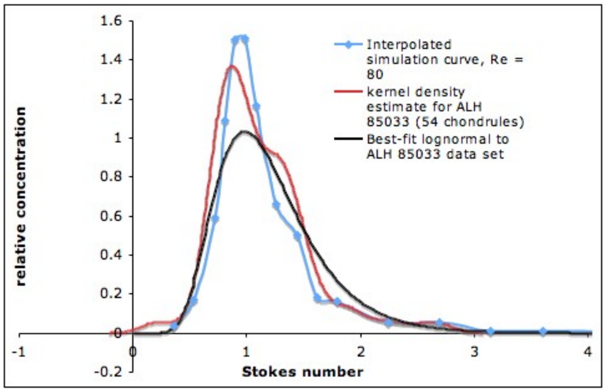

Figure 1: **Comparison of Chondrite Data Set, Best-Fit Lognormal, and Turbulent Concentration Curve**

location parameters. The test compares favorably with other tests with the same or similar properties [9]. We extended the analysis to test sorting by parameters other than the aerodynamic combination ρr. Table 1 shows the results of 21 comparisons (excluding the OTTA80301 data set–see below) for five possible sorting parameters.

Test 3: Finally, to test the TC hypothesis, we used Miller's jackknife test to compare chondrite data sets with three simulation data sets generated from the interpolated TC prediction curve, yielding 28 pairwise (observation vs. simulation) comparisons. To find the power and significance of this analysis, we then ran a series of Monte Carlo tests of paired simulation data sets with the same and shifted sample means, and a series of tests comparing simulation data sets with data sets drawn from various simple distributions.

sorting parameter	number of rejections
r	3
ρ	4
ρr	0
ρr^2	6
ρr^3	6

Table 1: Test 2 (sorting parameters)

Preliminary Results. In all tests we found that one chondrule sample (OTTA80301) behaves in an anomalous way, so we report results with and without its inclusion. This sample has by far the lowest fraction of measured data from the complete sample, opening the way for various systematic biases.

Using the Anderson-Darling test, we have shown that the eight chondrule sets as a group are not consistent with being drawn from a lognormal or Weibull distribution in r or ρr. Of the eight data sets, OTTA80301 has one of the two largest p-values for lognormality in r, and by far the largest p-value for lognormality in ρr.

We used Miller's jackknife test to show that the data sets are consistent (0 rejections out of 21 pairings) with being drawn from some chondrule distribution as a function of ρr (as would occur for an aerodynamic sorting process), if one particular data set (OTTA80301) is excluded. If OTTA80301 is included the group is not consistent (6 rejections out of 28 pairings, all involving OTTA80301) with being drawn from a distribution in ρr. We also showed that the data sets (whether or not OTTA80301 is excluded) are inconsistent with being sorted by r, ρ, ρr^2, or ρr^3.

Test 3 used Miller's jackknife test to show that the data sets are consistent (0 rejections out of 21 pairings) with being drawn from the TC curve, if the OTTA80301 data set is excluded. If the OTTA80301 set is included, the group is marginally consistent with being drawn from the TC curve (2 rejections out of 28 pairings). We have generated power tables against various alternative hypotheses for the Miller's jackknife test procedure. As expected, these tables indicate that the procedure is robust and maintains good power against most alternatives checked, but not every possible alternative.

Summary. Three statistical tests were carried out across seven chondrule samples drawn from six different chondrites. An eighth chondrule sample from a seventh chondrite was anomalous in all tests and, having a low measured fraction, presents a possibility of bias. Considered as a group, the samples are inconsistent with being drawn from lognormal or Weibull distributions. The samples are consistent with aerodynamic sorting (sorting by ρr) and inconsistent with sorting by r, ρ, ρr^2, or ρr^3. The samples are also consistent with being drawn from the specific TC distribution [2].

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