Introduction: In the last few years, 15 Super-Earths have been discovered [1], which have mass between 1-10 M\(_{\oplus}\) [2]. It is well-known, that the Earth-type planets have been differentiated and they have distinct spherical shells. The mantle of a differentiated terrestrial planet is divided into two parts as lower mantle and upper mantle, and its core might also be divided into a (liquid) outer core and (solid) inner core, if it has adequate physical and compositional properties.

Several physical parameters of planetary interiors are relatively well approximated by the equations of state (EOSs). The Vinet [3,4] and BME [5,6] EOSs are adequate analytical approximations for the extrapolations below 200 GPa.

The intermediate pressure range from 200 to 1000 GPa can be well described by the modified Thomas-Fermi-Dirac (TFD) EOS [7]. The purpose of this study is to derive theoretical mass-radius relations and detailed internal structure model for the chemically and mineralogically differentiated Earth-like compositional planets.

Model: I calculated the internal structure models of massive terrestrial planets for the case of 2.5 and 10 M\(_{\oplus}\). In this manner, I determined the density profile and temperature profiles as a function of the radius.

The selected chemical and mineralogical compositions are similar to that of Earth [8]. In my model, the inner core has Fe\(_{90}\), (Ni, Co)\(_{10}\), and the outer core composed of Fe plus FeS\(_{2}\) and FeO\(_{4}\). The core mass fraction (CMF) is 32.80 % [9] of the total mass. The upper mantle composed of olivine (Mg,Fe)SiO\(_{4}\) and its higher pressure variants (ringwoodite, wadsleyite); the lower mantle mainly composed of perovskite (MgSiO\(_{3}\)) plus wustite (Fe, Mg)O and in the zone of lowermost mantle: post-perovskite plus wustite. The post-perovskite compound is similar to that of Earth’s lowermost mantle [10]. The transition pressure between perovskite and post-perovskite phase is 125 GPa [10].

The depth-dependence of density is derived as:

\[
\frac{d\rho}{dr} = \frac{\rho(r)g(r)}{K_s(r)\rho(r)}
\]

(1)

Where \(r\) is the radius, \(K_s\) is the adiabatic bulk modulus which can be calculated with an EOS.

We considered the mineral phase transitions on different pressures, especially at the extremely high pressures. The pressure increases as a function of the decreasing radial change is the follow:

\[
\frac{dP}{dr} = -G \frac{M(r)}{r^2} \rho(r)
\]

(2)

Where \(G\) is gravitational constant, \(M\) is the planet mass. I have used Vinet, BME (below 200 GPa) and the modified TFD (above 1000 GPa) EOSs, and I applied their merged form which is the modified polytrophic EOS [11]. At the model calculations I considered the date of PREM [12].

The internal temperature values depend on the \(M\). Larger terrestrial planets have a more convective interior due to the higher temperature. The more intensive heat transport results in higher surface heat flux and thinner lithosphere. Therefore the terrestrial planets are more geologically active in case of larger mass. Consequently, the effective plate tectonics may be likely feature of the Super-Earths.

Furthermore, I provided a continental lithosphere model for the structural analysis. The mean continental lithospheric thickness is calculated using by lithosphere-seismic structure data (13). The lithospheric compound from the surface in the direction of radius is: silicate crust, peridotite, eclogite and olivine plus pyroxene. The continental lithospheric thickness (\(D_c\)) is numerically modeled with respect to parameters of dynamic lithosphere and geophysical properties. \(D_c\) can be approximated with the following relationship:

\[
D_c \approx \left[ \frac{l(\lambda C_p \rho)}{V_p} \right]^{1/2}
\]

(3)

Where \(l\) is plate length, \(\lambda C_p \rho = k\), which is the theoretically calculated thermal diffusivity for the continental type lithosphere and \(V_p\) is the plate velocity, respectively.

\(k\) is also computed by considering geothermal implications [13]. The aforementioned implications can be used to the more complicated calculations. Thus, the achieved parameters are suitable to geothermal modeling of planet interiors and preparation of temperature profile.

Results: I performed the model calculations and obtained scaling laws for total radius, core size and lithospheric thickness as a function of mass. The scaling law obtained for the total radius of 2.5 and 10 M\(_{\oplus}\) planets is \(R \propto M^{0.26}\). Whereas the mentioned core composition and the density-pressure relations can produce a relatively large exponent for the core size, which value is 0.255. The planetary structures have been modeled compared to the Earth. Moreover, the P-T profile and the density have also been theoretically calculated and their curves can be seen as Figures 1 and 2.
Radius and structural parameters are systematized in the Table 1. According to the pressure-temperature profile, the temperature of outer cores is higher than the melting temperature of Fe plus FeS and FeO alloys, therefore these are in liquid state. A similar composition yields liquid outer cores for all Earth-like compositional massive terrestrial planets. At the same time, the pressure/density conditions yield solid inner cores.

Figure 2.: Density profile for Super-Earths for the case of Earth-like composition and 2, 5, 10 Earth masses. The solid line shows the curve of 10 $M_E$ planet, dashed line is for 5 $M_E$ and dotted line is for 2 $M_E$ planet. The curve of Earth is also delineated (by relevant date: PREM; Dziewonski and Anderson [13]) for reference.

![Graph showing density profile](image)

I ascertained when I extended the planet mass the thickness of post-perovskite belt more significantly increased compared to other spherical shells due to the exponent of core size. Consequently, if the CMFs would be much less than that of Earth-like ratio the silicate mantle of Super-Earths is mostly composed of post-perovskite. The lithospheric thickness is inversely proportional to the planet mass. I computed the scaling law concerning the lithosphere, which prescribed by

$$D_L = D_0 (M/M_E)^{0.444}$$  \hspace{1cm} (4)

Manifestly, a few percent uncertainties will always derive from the inaccurate date in all models, because I cannot identify accurately the detailed composition of different spherical shells.

Summary: I expect that with 1-2% uncertainty in planet mass and radius it would be able to reasonably well determine not only main composition of iron/silicate planets, but conclude the substantial parameters of their interior structure. I believe that in the future the various internal structure models will be applicable as structural description of the Super-Earths.

References: