A GENERALIZATION OF BERNOULLI'S EQUATION TO CONVECTIVE VORTICES. N. O. Renno, D. G. Halleaux, F. Saca, S. Rogack, R. Gillespie, and Stephen Musko (renno@alum.mit.edu) Department of Atmospheric, Oceanic and Space Sciences, University of Michigan, Ann Arbor, MI 48109, USA

Introduction: Convective vortices ranging from small to large-scales are common features of planetary atmospheres. On Mars they play an important role on dust lifting and transport. Some of these vortices such as tornadoes and hurricanes are among the most damaging natural phenomena on Earth. Most of this damage is a consequence of strong jets along their spiral bands and eyewall. A recently proposed general theory for convective vortices predicts that pressure drops along these jets force a secondary circulation that produces the observed spiral bands [1]. This theory also sheds light on other basic features of convective vortices such as the formation of their eyewall and spiral bands. Here we show that an instrument capable of measuring the static and stagnation pressures, the wind vector, and the electric field vector proved the proposed theory, and in particular a generalization of Bernoulli's equation derived from it. This is important because this general theory for convective vortices sheds light on the physics of dust devils, waterspouts, tornadoes, and hurricanes, on Earth and beyond [1].

Theory: The maximum bulk intensity of convective vortices can be calculated assuming that they reach steady state [2]. In this case, the energy equation for a parcel of air follows from the dot product of the velocity vector with the equation of motion

$$d\left(\frac{1}{2}v^2 + gz\right) + \alpha dp = -\vec{f}\cdot d\vec{l} \qquad (1)$$

where v is the magnitude of the vector velocity, g the gravity acceleration, z the height above a reference level, α the specific volume, p the static pressure, $\vec{f}$ the frictional force per unit mass, and $d\vec{l}$ an incremental distance along the air parcel trajectory. Renno [1] showed that it follows from Eq. (1), the first law of thermodynamics, and a few minor approximations that

$$-(1-\gamma\eta)\frac{\Delta p}{\rho_s} + \gamma\eta\left[\left(c_p\Delta T + l_v\Delta q\right) + W_{irr}^{ab}\right] + \Delta\phi + \frac{\Delta v^2}{2} = 0 \qquad (2)$$

where γ is the fraction of the mechanical dissipation of energy occurring at the heat input branch of the circulation, η the thermodynamic efficiency, $\left(c_p\Delta T + l_v\Delta q\right)$ the change in enthalpy, W_{irr}^{ab} the irreversible work of expansion, $\Delta\phi$ the change in potential energy, and $\frac{\Delta v^2}{2}$ the change in kinetic energy, between the vortex radius of influence at point a, and a point b in the updraft [1]. Eq. (2) is a generalization of Bernoulli's equation to convective circulations. It provides insights into the physical processes causing pressure changes along the trajectory of convecting air parcels.

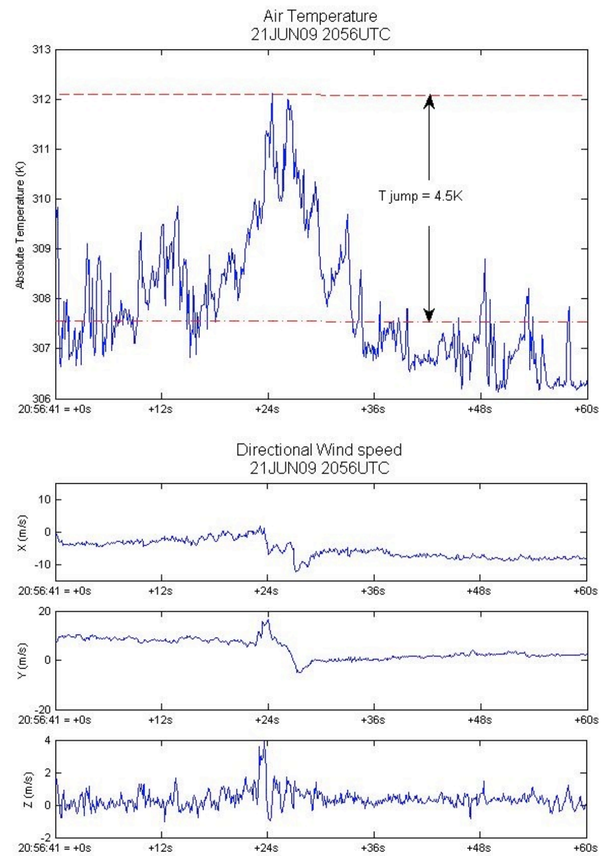

Fig. 1a. Temperature and windspeed measurements on a large dust devil in Nevada.

Eq. (2) predicts a decrease in static pressure (first term) with increases in kinetic energy (last term). This implies an anti-correlation between static and stagnation pressures (or windspeed). We postulate that this has not been observed before because usually the pressure is not properly measured. This occurs because pressure intakes are usually not properly aligned with the flow. The main goal of our field measurements was to test Eq. (2) by making careful measurements of the static and stagnation pressures. In addition, the temperature, wind and electric field vectors were measured to give insights on the physics of dust devils.

Measurements: It follows from Eq. (2) that temperature perturbations of ~4.5 K such as that indicated in Fig. 1a, causes maximum pressure perturbations of ~1.1 Pa for dust devils with $\eta \sim 0.05$ or about 5 km deep [1]. This is consistent with our observations and measurements in large dust devils.

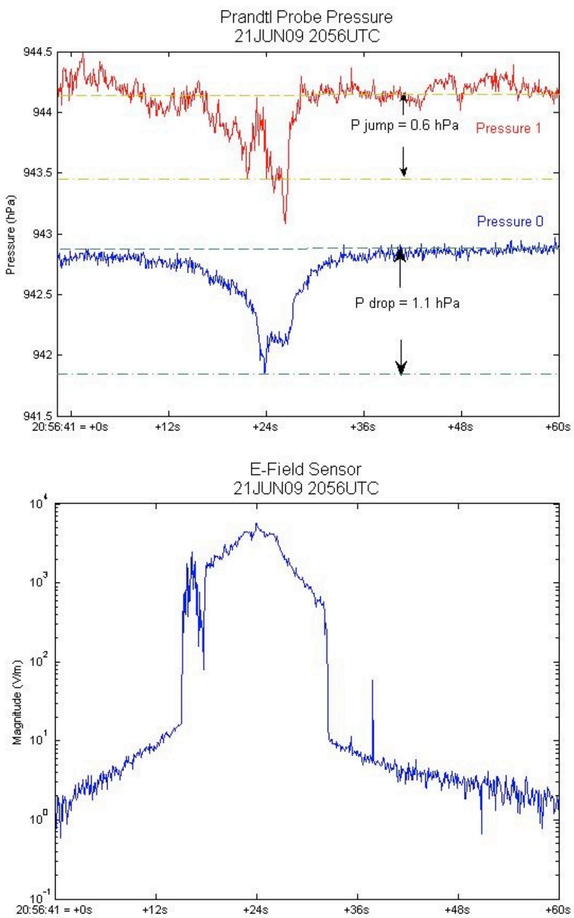

Fig. 1b. Static pressure (red), stagnation pressure (blue), and electric field values on the dust devil of Fig. 1a. The abrupt increase in the electric field indicates the passage of the dust wall. The electric field peaks in a dusty spiral band. As predicted by Eq. (1), there is an anti-correlation between static and stagnation pressures in this region, around 24 s. Fig. 1a also indicates that the vertical component of the velocity peaks in this region, a consequence of the secondary circulation forced by the drop in static pressure.

Discussion: What concentrates dust and clouds into a ring (the eye wall) around the center of convective vortices such as dust devils and tornadoes? What causes the formation of spiral bands? Eq. (2) indicates that increases in the kinetic energy along jets of air flowing into the vortex causes a decrease in static pressure. This, in turn, can cause adiabatic cooling and condensation, and forces a secondary circulation that focuses dust and cloud particles in the region of maximum wind. Near the surface, the convergence cuased by this secondary circulation forces updrafts that might trigger convection and lift debris such as dust particles.

Fig. 2. Image of a tornado showing the funnel cloud and a well-defined spiral band. Eq. (2) predicts that decreses in static pressure in regions of maximum wind form hollow vortices and spiral bands on dust devil, waterspouts, tornadoes and hurricanes [1]. **Credit:** Copyright by Edi Ann Otto.

Conclusions: Our measurements proved a general theory for convective vortices [1] and the suggestion that it explains the formation of hollow vortices and spiral bands on Earth, Mars and beyond. In order to properly study these systems, the static and stagnation pressures must be measured more carefully that previously done.

Aknowledgments: This research was partially funded by NASA and NSF.

References: [1] N. O. Renno (2008) *Tellus, 60A, 688–699*. [2] N. O. Renno, M. L. Burkett, and M. P. Larkin (1998) *J. Atmos. Sci.*, **55**, 3244-3252.