Lunar obliquity during a Cassini-state transition. B.G. Bills[1], W.B. Moore[2], M.A. Siegler[2], and D.A. Paige[2]
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Introduction: The spin pole of the Moon is tidally damped. As a result, it currently occupies a Cassini state [1,2]. That is, the obliquity is set to a value at which the spin and orbit poles remain coplanar with the ecliptic pole, during each precession cycle. The current damped obliquity is only 1.6 degrees, which is small enough that some regions inside polar craters are permanently shadowed [3,4,5]. During the Moon's orbital evolution away from Earth, the spin pole transitioned from one Cassini state to another, and went through a brief period of very high obliquity [6]. In order to model the near-surface temperature variations during these times of high obliquity, and determine the fate of any volatiles trapped there, we first need to develop improved models this dynamic spin pole transition.

We will briefly recount the main features of the orbit and spin evolution of the Moon, as they pertain to this episode of high obliquity. The main focus of our study is the dependence of the spin pole transition upon variation in moments of inertia of the Moon during the transition.

Orbit precession: The main driver in this story is tidal energy dissipation within Earth's oceans. This has caused the Earth-Moon orbit to increase in size [7,8,9]. There are associated variations in orbital eccentricity and inclination, but the main effect of interest here is a significant change in the rate of orbit plane precession. The orbit plane precesses in response to two torques, one from the oblate Earth and another from the Sun. The solar torque, which makes the orbit pole precess about the ecliptic pole, increases in strength with increasing distance from Earth. The oblate figure torque, which makes the orbit pole precess about Earth's spin pole, decreases with increasing distance. The current precession period is 18.6 years, but it reached a maximum value of roughly 80 years, at the hand-off from Earth-control to Sun-control, at roughly 1/3 the present Earth-Moon distance [7,8]. See Figure 1, below.

Spin precession: The lunar spin pole precesses at a rate which depends both on distance from Earth, and on the difference between the polar and equatorial moments of inertia. If the spin pole and orbit pole unit vectors are denoted $\hat{s}$ and $\hat{n}$, then the spin pole precesses according to

$$\frac{d\hat{s}}{dt} = \left(\alpha(\hat{n} \cdot \hat{s}) + \beta\right)\left(\hat{s} \times \hat{n}\right) \qquad (1)$$

where the rate constants α and β are proportional to the orbital mean motion n, and are given explicitly by

$$\alpha = \frac{3}{2}n\left(\frac{C - (A + B)/2}{C}\right) \qquad (2)$$

and

$$\beta = \frac{3}{2}n\left(\frac{B - A}{4C}\right) \qquad (3)$$

where the principal moments of inertia are A < B < C.

Early in its history, departures from spherical symmetry in the lunar mass distribution were close to hydrostatic [10]. A more rapid rotation would make the Moon more oblate, while a larger imposed tidal potential would tend make it a prolate spheroid, with symmetry axis aligned with the Earth-direction.

It is not known when, in its thermal and orbital history, the Moon started departing from hydrostatic equilibrium, but it is known that the current degree two gravity field of the Moon is far from hydrostatic equilibrium [11,12,13].

Gravity models: The principal moments of inertia are related to the two non-zero terms (J_2 and $C_{2,2}$) in the spherical harmonic expansion of the degree two gravity field via

$$J_2 M R^2 = C - (A + B)/2 \qquad (4)$$

and

$$C_{2,2} M R^2 = (B - A)/4 \qquad (5)$$

Current values are [14] $J_2 = (203.67 \pm 0.07) \times 10^{-6}$, and $C_{2,2} = (22.19 \pm 0.01) \times 10^{-6}$. In contrast, the hydrostatic values are $J_2 = (9.38) \times 10^{-6}$ and $C_{2,2} = (2.83) \times 10^{-6}$.

We consider several models for past variations in the moment differences. In the simplest model (constant value), we just assume that the current values have always been applicable. In the next most complex model (constant bias), we assume that the hydrostatic response has always been present, but that it is augmented by a value which makes the sum equal to the currently observed values. We also include models in which the augmentations, or bias values, are either linear or quadratic functions of Earth-Moon distance.

In each case, the rotational potential is a function of the Earth-Moon distance, as we assume synchronous rotation throughout. The tidal potential, in contrast, is a function of both the Earth-Moon distance and the mean obliquity.

Obliquity variation: To occupy a Cassini state, the obliquity must be such that the angular rate of spin pole precession, and length of the precessional path, are adjusted so as to match the orbit pole precession period. The criterion for steady co-precession of a tri-axial synchronous rotator can be written as [1,2,6]

$$\eta \sin[i - \varepsilon] = \left(\alpha \cos[\varepsilon] + \beta\right) \sin[\varepsilon] \quad (6)$$

where η is orbit pole precession rate, ε is obliquity, and i is orbital inclination.

This equation has either two or four real roots when solved for obliquity, depending upon the values of the input parameters (α, β, η, i). At an early point in lunar orbital evolution, the parameters were such as to allow four solutions. The Moon's spin pole presumably first damped into the lowest obliquity Cassini state, known as state S1. However, as the orbit evolved, two of the Cassini states (S1 and S4) merged and disappeared. The Moon then transitioned to state S2, where it is today. Figure 2 illustrates these obliquity variations, under the simplifying assumption that the gravity field remained constant.

We will present obliquity histories in which the gravity field follows the more realistic (but still somewhat contrived) recipes described above.

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lunar orbit precession period

lunar obliquity variation