

A SCALED MODEL DESCRIBING THE COMPRESSIVE STRENGTH OF GEOLOGIC MATERIALS.

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The vast majority of compressive failures of brittle solids involve *both* significant amounts of crack nucleation (from pre-existing flaws or defects) and significant amounts of crack propagation and crack interactions. A new ansatz for high-rate compressive failure was recently developed by Paliwal and Ramesh [1] based on real-time ultra-high-speed visualization experiments [2] coupled with theoretical investigations of massive brittle failure. In brief, that work showed that the dynamic compressive failure process is controlled by the interactions of three terms: the initial defect distribution, crack growth dynamics and crack-crack interactions, and the coupling of these three terms with the superimposed rate of loading. That model describes the increase in the strength that is often observed in rocks subjected to uniaxial compression at high strain rates [3, 4, 5]. This work develops a model that captures the behavior of brittle solids in an appropriately scaled form [6].

We begin by identifying the critical length scales and timescales associated with the physical problem of a brittle solid containing a pre-existing distribution of rectilinear flaws and subjected to dynamic loading. The pre-existing flaw distribution is described in terms of a flaw density η (the number of flaws per unit area), and a pdf $g(s)$ of the flaw size s . This results in the introduction of two length scales into the system: the mean flaw size \bar{s} and the average flaw spacing $\eta^{-1/2}$. Since information in an elastic material propagates at a finite velocity, these two length scales also result in two timescales, associated with the communication time between crack tips from a single flaw, and the communication time between flaws. Crack initiation is controlled by the fracture toughness, K_{IC} , and the dynamics of subsequent crack growth introduces another timescale into the problem, that of the time it takes for a wing crack to grow to span the average distance between flaws. The interaction between these timescales and the timescale associated with the applied load determines the response of the material as a function of the rate of loading.

This compressive strength model thus has five material parameters: η , \bar{s} , E (the Young's modulus), c , K_{IC} , which together control the strength σ_f of the material at a given strain rate $\dot{\epsilon}$. These parameters span a significant range of values in rocks. Model simulations were conducted using the framework developed by Paliwal and Ramesh [1] for a range of material properties typically observed in rocks, and a range of strain rates from 10^2 - 10^6 s⁻¹; each simulation provides a peak

value of the computed stress, which is defined as the compressive strength, σ_f . The range of possible combinations of material parameters results in a very large range in the predicted compressive strengths and in the dependence of that strength on the strain rate; such a range is in fact observed in geophysical materials [3, 4, 5, 7]. We show that these results can be collapsed into a specific scaled form.

Scaled Behavior of Brittle Solids in Compression: A scaled response function can be developed for all of the available results by defining a characteristic stress and a characteristic strain rate in terms of the microstructural variables and the length and time scales in the problem. The physics of the problem and the non-dimensional combination [8] of the key variables allows us to determine a characteristic stress σ_o as

$$\sigma_o = \frac{K_{IC}\eta^{1/4}}{\bar{s}/\eta^{-1/2}}. \quad (1)$$

Here the numerator in the first expression is essentially the far-field stress associated with a crack of length equal to the flaw spacing, while the denominator is the ratio of the average flaw size to the flaw separation. This characteristic stress can be thought of as the stress required to generate a wing crack sufficiently long to bridge the flaws for any given flaw density and flaw size distribution. The timescales in the problem provide a characteristic strain rate $\dot{\epsilon}_o$ defined by

$$\dot{\epsilon}_o = \frac{c}{\bar{s}} \frac{K_{IC}\eta^{1/4}}{E}, \quad (2)$$

where the first term represents the reciprocal of the communication time along the initial flaw, while the second term represents the far-field strain associated with a crack of length equal to the flaw spacing. Normalizing the failure stresses predicted by the Paliwal-Ramesh model [1] by the characteristic stress, and normalizing the applied strain rate by the characteristic rate collapses the data around a single curve. The equation of this curve has the form,

$$\sigma_f/\sigma_o = 1.1 + (\dot{\epsilon}/\dot{\epsilon}_o)^{2/3}. \quad (3)$$

The 2/3 exponent provides an excellent fit, and represents the scaling that is associated with the competition between the surface mechanism (fracture) and a volumetric quantity (the stored strain energy). This scaled model compares very favorably with experimental observations of the rate-dependent compressive strength of several geologic materials as shown by Figure 1. The solid line in Figure 1 represents the curve described by equation (3), and the light gray

band represents the scatter that is observed in the normalized simulated results. The experimental data are represented by the symbols. It is apparent that the scaled function represented by equation (3) is capable of describing a number of geological materials in compression. Table 1 presents the characteristic stress and the characteristic strain rate for each material and each data set presented in Figure 1. The ability of the model to describe the behavior suggests that the important micromechanics has been captured. The scaled results show that the compressive strength remains nearly constant below a transition strain rate, but a rapid increase in strength develops as the strain rate is increased above the transition (characteristic) strain rate.

While not shown explicitly in the scaled representation, the damage in the material also increases substantially with the rate of loading [1]. The increases in compressive strength with increasing rate is important in the early stages of planetary impact, and the associated accumulation of internal damage the influences the apparent tensile strength of the material during the subsequent modification phase. Such stress–path evolution of the strength may, for example, play a role in situations such as the unusual simple-to-complex transition observed in craters on Mercury (where the impact velocities and thus strain rates are high). The above analysis provides a micromechanics based constitutive model for describing failure strength as a function of strain rate that compares well with experimental data for a wide range of brittle materials. The resulting material model is expressed in a simple functional form making it suitable for incorporation into more general analysis codes (e.g., various finite element packages). Furthermore, the ability of the model to capture the physical behavior of brittle materials under uniaxial stress leads us to believe that the micromechanics based analysis used here may be applicable to more general states of stress (e.g. confined compression) that are often encountered in planetary impact problems.

References:

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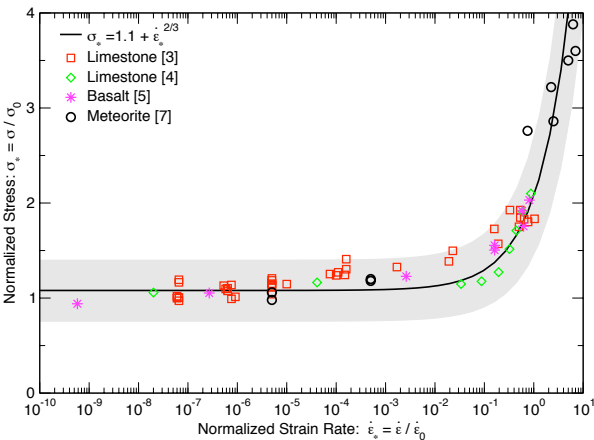


Figure 1. Comparison of the normalized model with experimental data for several geological materials.

Table 1. Characteristic Stresses and strain rates for experimental data.

Material	σ_0 (MPa)	$\dot{\epsilon}_0$ (s ⁻¹)
Limestone [3]	65	200
Limestone [4]	270	5000
Basalt [5]	200	1000
Meteorite [7]	50	200