Influence of tidal dissipation upon co-precessing spin and orbit poles

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Introduction: We discuss the influence of tidal energy dissipation upon the orientation of the spin pole of a satellite or planet, relative to its orbit pole. We also consider several specific cases in which dissipation rates may be high enough to have measurable influence on spin pole orientation

Background: The obliquity of a body is the angular separation between its spin and orbit poles. It is of interest both for cartographic and geophysical purposes. Knowledge of the obliquity is needed for characterizing the orientation of a body [1], it is an important factor in controlling planetary insolation patterns and associated climate change [2, 3], and it can also be diagnostic of internal structure [4].

If the body of interest were moving along an orbit with a fixed orbit pole, then tidal energy dissipation would simply damp the obliquity, and drive the spin pole into alignment with the orbit pole. In that case, the only possible value for damped obliquity is zero. In the more interesting case where the orbit pole is itself precessing, at a fixed inclination and steady angular rate about an inertially fixed invariable pole, dissipation drives the spin pole into a configuration, known as a Cassini state, in which it co-precesses with the orbit pole. That is, in a reference frame fixed in and precessing with the orbit plane, the spin pole, orbit pole, and invariable pole maintain fixed positions. Such steady co-precessional configurations are known as Cassini states [5, 6], in honor of G.D. Cassini, who first noted, in 1693, that the Moon behaves that way.

In most previous treatments of the dynamics of spin pole precession and Cassini states [5, 6, 7, 8], dissipation plays an ironic role. It is assumed to be sufficient to drive the spin pole to a steady state, but insufficient to modify that state. In contrast, we explicitly consider the role of finite dissipation in modifying the geometry of the damped state. In previous analyses of the dissipation-free Cassini state, it has been shown that the spin pole $\hat{s}$ remains in the plane defined by the orbit pole $\hat{n}$ and invariable pole $\hat{k}$. In contrast, we find that dissipation moves the damped spin pole out of the plane defined by $\hat{n}$ and $\hat{k}$, by an amount which increases with increasing dissipation. This has the important implication that, if the spin pole orientation of a body can be determined, and it can be established that the spin pole is fully damped, then the out-of-plane angle will be diagnostic of the dissipation rate.

Precession: We now examine the explicit form of the steady precession state. The orbit pole $\hat{n}$ is inclined to the invariable pole $\hat{k}$ by an angle i, and precessing about it at rate γ. The orbit pole thus moves according to

$$d\,\hat{n} / dt = -\gamma\left(\hat{k} \times \hat{n}\right)$$

while maintaining a fixed inclination to the invariable pole

$$\hat{n} \cdot \hat{k} = \cos[\, i\,]$$

In addition, the spin pole $\hat{s}$ precesses about the instantaneous orientation of the orbit pole $\hat{n}$, so that its motion, in an inertial frame, is governed by the differential equation

$$d\,\hat{s} / dt = F[\hat{n}, \hat{s}]$$
$$= (\alpha\ (\hat{n} \cdot \hat{s}) + \beta\,)\,(\hat{n} \times \hat{s}) + \delta\,(\hat{n} - (\hat{n} \cdot \hat{s})\,\hat{s})$$

To see this motion in an orbit fixed frame, we write it as

$$d\ \hat{s} / dt = F\,[\hat{n}, \hat{s}] + \gamma\,(k \times s)$$

where α and β are precession rate parameters and δ is the damping rate. For a synchronous rotator, with principal moments of inertia A < B < C, the inertial rate parameters are [6, 8]

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \frac{3n}{8C}\begin{bmatrix}4(C-A)-(B-A)\\ (B-A)/2\end{bmatrix}$$

where **n** is the orbital mean motion. The damping rate is given by [9]

$$\delta = \frac{3}{2}\,n\left(\frac{k}{Q}\right)\left(\frac{M\,R^2}{C}\right)\left(\frac{R}{a}\right)^3$$

where M and R are mass and mean radius, while k is the tidal Love number and Q is the tidal quality factor.

In the absence of dissipation (δ = 0) the Hamiltonian for this motion is [6,8]

$$H = (\alpha / 2)\left(\hat{n} \cdot \hat{s}\right)^2 + \beta\left(\hat{n} \cdot \hat{s}\right) + \gamma\,(\hat{k} \cdot \hat{s})$$

This scalar function is conserved along spin pole trajectories. Geometrically, it is a parabolic cylinder, with axis perpendicular to the plane defined by $\hat{n}$ and $\hat{k}$. Intersections of this parabolic cylinder with the unit sphere

$$\hat{s} \cdot \hat{s} = 1$$

determine the trajectories of the spin pole. If the energy is such that the Hamiltonian surface intersects the unit sphere in a point of external tangency, the trajectory collapses to a single point, which is a Cassini state.

Dissipative fixed points: For spin pole precession with dissipation, the Hamiltonian is not conserved and an alternative characterization of the fixed points is needed. This can be obtained by examining the null-clines [10], or curves along which the zonal or meridional components of precessional velocity vanish. In the Hamiltonian approach, we examine possible trajectories of the spin pole, and find special cases where they reduce to a fixed point. In contrast, the null-cline approach examines locations at which the spin pole cannot move in either latitude or longitude. Fixed points are then found as intersections of those curves.

We thus take

$$\hat{s} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin[\theta]\cos[\varphi] \\ \sin[\theta]\sin[\varphi] \\ \cos[\theta] \end{bmatrix}$$

where the x- and y-axes lie in the orbit plane, with the y-axis defined by the intersection of the orbit plane with the plane defined by $\hat{n}$ and $\hat{k}$. For the spherical coordinates, θ is colatitude and φ is longitude, measured from the direction of the x-axis.

When this representation of the spin pole is substituted into the non-dissipative equation of motion, we find that the velocity vanishes when x = 0 and

$$y = \frac{c\,z}{a\,z + b}$$

where a = α, b = β+γ cos[i], and c = γ sin[i]. This latter constraint corresponds to a rectangular hyperbola in the y-z plane, with center at $\{y_c, z_c\}$={+c/a, -b/a}. One branch of the hyperbola passes through the origin {y,z}={0,0}, and that branch always has an intersection with the unit sphere. Depending upon the parameters, the other branch may, or may not, intersect the unit sphere. In this non-dissipative case, fixed spin poles, or Cassini states, correspond to intersections of 3 surfaces: the y-z plane, the hyperbolic cylinder, and the unit sphere.

When we include dissipation, there is a change in only one of the 3 surfaces whose intersections characterize fixed points. The angular velocities of the spin pole become

$$v_\theta = \frac{d\theta}{dt} = c\cos[\varphi] - \delta\sin[\theta]$$

and

$$v_\varphi = \sin[\theta]\frac{d\varphi}{dt} = (a\cos[\theta] + b)\sin[\theta] - c\cos[\theta]\sin[\varphi]$$

Note that dissipation only changes the meridional velocity.

The $v_\theta = 0$ null-cline lies on the intersection of the unit sphere with a circular cylinder

$$(x - R)^2 + y^2 = R^2$$

whose radius is given by

$$R = c/(2\delta)$$

In the limit of no dissipation, the radius diverges and the circle becomes a line coincident with the y-axis, thereby reverting to the non-dissipative geometry seen earlier The main effect of dissipation, in this context, is to move the damped spin pole out of the y-z plane.

The two null-clines intersect at points whose colatitude θ satisfies the equation

$$c^2 = ((a\cos[\theta] + b)\tan[\theta])^2 + (\delta\sin[\theta])^2$$

As with the dissipation-free case, the number of possible Cassini states changes with the values of input parameters. There can be 2, 3, or 4 different Cassini states at each point in the parameter space. During the Moon's orbital evolution, it transitioned from 4 to 2 possible states, and experienced a short period of very high obliquity, while in transit from one damped state to another [8]. Absent dissipation, the transition occurs when [7]

$$(b/a)^{2/3} + (c/a)^{2/3} = 1$$

When dissipation is included, the transition criterion changes to a much more complex form. Changing the dissipation rate can either create or destroy fixed points, depending upon the other input parameters.,

Applications: In application to the Moon, where lunar laser ranging data provide very tight constraints [11] there is evidence for some dissipation [12, 13, which is manifest as a small rotation of the spin pole away from the expected non-dissipative position.

For Saturn's largest satellite Titan, Cassini mission radar mapping has allowed a recent determination of the spin pole orientation [14]. It is close to, but not exactly in a conventional Cassini state. In that case, it is not yet clear whether there is too little damping, or too much. That is, the slight displacement from a Cassini state could be a manifestation of incompletely damped spin pole motion, with a finite mode of free precession persisting. Alternatively, it could reflect sufficient damping to displace the Cassini state from the non-dissipative location.

In other cases, where dissipation rates are known to be large, such as Io [15, 16] and Enceladus [17, 18], measurements of the orientation of the spin pole will help quantify these rates.

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