Lava flow dynamics driven by temperature-dependent viscosity variations.

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Background

Pahoehoe lava flows dominate terrestrial basaltic lavas throughout subaerial and submarine environments [1] on the Earth and other planetary bodies [2]. These landforms grow via the injection of thin lava flows (10-50cm thick) beneath an insulating crust. At the interface between the crust and the flow, a more viscous layer of lava cools and accretes, which causes the crustal thickness to increase, with buoyancy forces enhancing the uplift (as the crust is less dense than the hot lava). Initially flowing as a sheet with a uniformly distributed liquid lava core, terrestrial pahoehoe lava flows develop thermally and mechanically preferred flowpaths beneath an insulating crust [3] which can merge and mature, forming lava tube systems [4].

While the inflation mechanism and subsurface lava tube formation allows for more efficient delivery of small pulses of lava from the source vent to the flow front [5], these processes greatly complicate efforts to estimate flow duration, lava rheology, and overall flow evolution from surface morphology. We aim to mathematically model the mechanisms by which preferred lava flowpaths can develop within initially uniform thin sheets. In particular, we focus on dynamically-driven 'fingering' caused by large viscous variations (Figure 1), which can arise from small differences in temperature and (related) crystallization or volatile content. Fingering dynamics have been observed in the lab [3, 6] and have been used to explain temporal [7] and spatial [8] oscillation within lava flows.

Methodology

We investigate 2D profiles of channelized flow over a horizontal plane (Figure 1; the (x,z)-plane or the (x,y)-plane, where x is the along-flow axis). Due to a lava flow's geometry (flow depth $\ll$ length and width [11], a Hele-Shaw approximation), we approximate flow as laminar thin flow, which yields a modified Darcy's law (EQN 1) relating the pressure gradient to the velocity. Heat advects and diffuses within the flow (EQN 3) and is lost to the cooler walls as the wall rock heats up and insulates the magma. The thermo-viscous relation is approximated using Nahme's exponential law (EQN 4).

Non-dimensionalized system equations are:

$$\begin{aligned} \nabla \cdot U &= 0 \\ \nabla P &= \mu U && (1) \\ \text{(equivalently, } 0 &= \nabla \cdot (\mu \nabla \psi)) && (2) \\ T_t + u \cdot \nabla T &= T_{zz} && (3) \\ \mu &= \exp(\beta(1-T)) && (4) \end{aligned}$$

where U is flow velocity, P is the pressure, ψ is the associated streamfunction, μ is the viscosity, and T is the fractional temperature (between T_{hot} and T_{cold}). β is a free parameter that controls the strength of the viscosity's temperature dependence ($= \ln(\mu_{\text{cold}}/\mu_{\text{hot}})$).

Our model is very similar to the model presented in [8] for vertical lava flow through a conduit, with the primary differences found in the boundary conditions (in particular, the temperature of the channel walls) and the flow orientation. Fingering dynamics were observed in these prior studies, in which flow became focused into hotter (and thus less-viscous) regions. The threshold at which fingers appeared depends on β and the influx velocity (u_{in}).

Analysis

The steady-state solution for our system is a flow with constant temperature and a uniform velocity. Beginning with this state, we assess the linear stability around a small perturbation:

$$\begin{aligned} T &= T_0 + \theta, \text{ where } T_0 = \text{ constant} \\ \psi &= S_0 + \phi, \text{ where } S_0 = u_{\text{in}} z + C \end{aligned}$$

The perturbations (ϕ and θ) thus evolve via:

$$\begin{aligned} \theta_t + u_{\text{in}} \theta_x &= \theta_{zz} \\ \phi_{zz} + \phi_{xx} &= -\beta u_{\text{in}} \theta_z \end{aligned}$$

i.e., the temperature variations evolve via simple advection-diffusion and the streamfunction perturbation evolves via a Poisson relation. Additionally, the instability threshold wavenumber scales with βu_{in}. Note that if $\beta = 0$ (constant viscosity case), then the perturbations evolve via the same equations as the steady-state flow (EQN 3 and 2).

We are also numerically simulating the evolution of flow within the channel – the algorithm iterates between

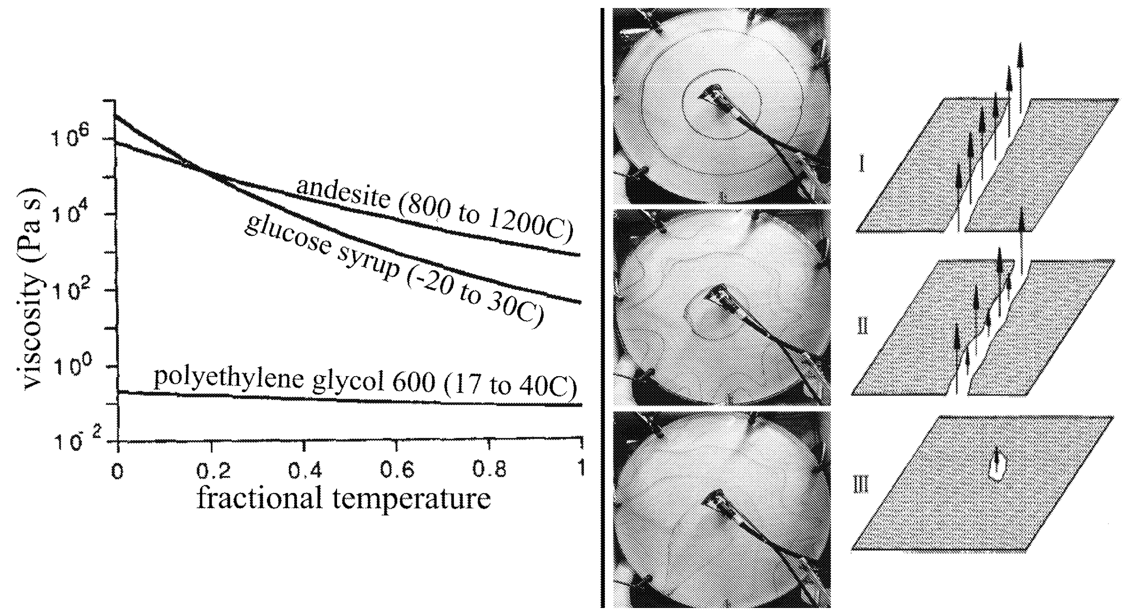

Figure 1: The plot shows the strong temperature-dependence of viscosity of several fluids, taken from [9]. The abscissa is fractional temperature within the ranges specified for each fluid. The photographs show the formation of Saffman-Taylor fingering in warm Karo syrup that is cooled along the bottom of the horizontal Hele-Shaw cell, which are qualitatively analogous to fissure eruption stages, shown in the diagram: initial flow may be uniform (top), but small flow rate differences are enhanced by temperature/viscosity gradients to create fingers of preferential flow (middle). Over time, flow may evolve to contain one large stable flow-finger with the rest of the fluid nearly stagnant (bottom). The experiment images and fissure diagram were taken from [10].

evolution of the temperature field, given the streamfunction (ψ) field, and the streamfunction evolution, given the temperature (T) field. The channel walls have specified temperatures which may differ on the two sides (leading to asymmetric heat transfer) or may evolve (i.e., as the insulating crust thickens). The influx flow is of specified temperature and velocity. Free-slip conditions are assumed along the channel walls, as we neglect the details of boundary layer flow close to the wall.

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