3-D INVERSION OF THE GRAVITY DATA ON THE MOON. Q. Liang[1, 2], C. Chen[1] and Y. Li[2], [1]Institute of Geophysics and Geomatics, China University of Geosciences, 388 Lumo Road, Wuhan 430074, China, qingliang.cug@gmail.com, chenchao@cug.edu.cn, [2]Center for Gravity, Electrical & Magnetic Studies (CGEM), Department of Geophysics, Colorado School of Mines, Golden 80401, USA, ygli@mines.edu.

Introduction: Gravity data has been used to estimate the lunar crustal thickness, assuming that the crust and the mantle have constant density so the gravity anomalies are only produced by the variations of interface between the crust and the mantle, e.g., [1-3]. However, the density structures in the crust or mantle are less discussed. Alternatively, the inversion of gravity data can retrive the underground density distribution. It has been demonstrated that the three-dimensianal (3-D) inversion of gravity data is capable of retrieving density structures in resources exploration on the Earth [4]. We may thus apply this method to the lunar gravity anomaly and recover the density structures beneath mascon basins or the lateral density heterogeneities in crust and mantle. However, if considering the gravity data observed on sphere in large scale, there are limitations in the previous inversion because the classical methods are based on the Cartesian coordinates system.

Inverse Problems in Spherical Coordinate System: We use here a geographical spherical coordinate system described by longitude, latitude and radius of a sphere (Fig. 1). Based on a mesh model gridded by longitude and latitude, the gravitational attraction δg induced by density of a tesseroid cell at the observation location r' is written in the form of Newton's integral [5],

$$\delta g(r',\varphi',\lambda') = \rho \cdot G \int_{\lambda_1}^{\lambda_2}\int_{\varphi_1}^{\varphi_2}\int_{r_1}^{r_2} \frac{r^2 (r' - r\cos\psi)\cos\varphi \, dr \, d\varphi \, d\lambda}{\ell}, \quad (1)$$

where G is universe gravitational constant, ρ is density, λ, φ, r are longitude, latitude and radius of the geometric center Q in tesseroid, respectively. λ', φ', r' are the ones of observation location P. Ψ is defined as the spherical angle between P and Q. l is distance between the mass center and the observation point. $\ell = \sqrt{r'^2 + r^2 - 2r'r\cos\psi}$, and

$\cos\psi = \sin\varphi'\sin\varphi + \cos\varphi'\cos\varphi\cos(\lambda - \lambda')$.

Numerically, the zero-order approximation of gravitational attraction [5], eqs. (1), is written as

$$\delta g(\lambda',\varphi',r') = \rho \cdot G r^2 \cos\varphi \Delta r \Delta\varphi \Delta\lambda \frac{r' - r\cos\psi}{(\sqrt{r'^2 + r^2 - 2r'r\cos\psi})^3}, \quad (2)$$

where $\Delta\lambda$, $\Delta\varphi$, Δr are the 3-D sizes of a Tesseroid. If we let $\boldsymbol{K}$ represents a kernel function, there is linear relationship between gravitational attraction (data) and density (model), i.e., $\delta g = \boldsymbol{K}\rho$.

Besides of the kernel function, the inverse problem in spherical coordinate system is also different in the model objective function. Generally, we attempt to fit the observed data but require that the model be relatively smooth in three spatial directions according to Backus-Gilbert theory [6-7]. Thus, due to the gradient operator in spherical coordinate, i.e.,

$$\nabla = \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\varphi \frac{1}{r}\frac{\partial}{\partial \varphi} + \vec{e}_\lambda \frac{1}{r\cos\varphi}\frac{\partial}{\partial\lambda}, \quad (3)$$

we can generate a new model objective function as

$$\begin{aligned}\phi_m(\rho) = {} & \alpha_s \int_V [w(r)(\rho - \rho_{ref})]^2 dv \\ & + \alpha_r \int_V \left[\frac{\partial w(r)(\rho - \rho_{ref})}{\partial r}\right]^2 dv \\ & + \alpha_\varphi \int_V \left[\frac{\partial w(r)(\rho - \rho_{ref})}{r\,\partial\varphi}\right]^2 dv \\ & + \alpha_\lambda \int_V \left[\frac{\partial w(r)(\rho - \rho_{ref})}{r\cos\varphi\,\partial\lambda}\right]^2 dv\end{aligned}, \quad (4)$$

where α_s, α_λ, α_φ, and α_r are the coefficients that weight different components in the objective function. The $w(r)$ is a depth weighting function [4] but modified in the new coordinate. The unit volume $dv = r^2 \cos\varphi \, dr \, d\varphi \, d\lambda$.

Generally, a data misfit function is defined as

$$\phi_d = \sum_{i=1}^{N}\left(\frac{d_i^{obs} - d_i^{pre}}{\sigma_i}\right)^2, \quad (5)$$

where the σ_i is the standard deviation of the ith data noise. Eventually, the inversion aims to minimize the total objective function

$$\phi(m) = \phi_d + \mu\,\phi_m. \quad (6)$$

In this function, μ is a regularization parameter which is used to trade off the data misfit and model complexity. An apposite μ should be selected by the Tikhonov curve [4]. The associated solution, therefore, provides interpretable results. After establishing objective function, we use conjugate gradient method to sovle the inverse problem. Several synthetic examples have been successfully examed. Here, we just apply this method to the lunar gravity data.

Application to the Lunar Gravity Data: The lunar gravity data we use is the Bouguer gravity anomaly at 10km height [8], calculated from gravity model SGM100h [9] and Chang'E-1 topography model CLTM-s01 [10]. In general, lava fills in the mascon basins and uplifted mantle beneath the basins are suggested to jointly produce the positive gravity anomalies with high amplitude. It is difficult to separate them into different components. We simply use tendency analysis to fit the regional anomalies with polynomial, and consequently separate the local anomalies. We assume the model space only occupies the depth from 0 km to 100 km. Firstly, the inversion of the gravity anomalies in Mare Serenitatis was carried out. Then we inverted the global data to recover the 3-D density distribution above 100 km depth.

Results show that the high density anomaly beneath Mare Serenitatis concentrates in southeast (Fig. 2), meaning that the distribution of density is not uniform laterally. The inverted result (Fig. 3) from global data presents the similar density structures beneath the mascon basins with that from regional data. Both the inversions yield the high density anomalies concentrating at the depth of 20-50 km. Furthermore, the major mascons seem to be connected by high density anomalies (Fig. 3). For the Feldspathic Highlands Terrane (FHT), the negative density anomaly also distributed at the same depth, although it has lateral span of 2300 km. The density structure beneath the South Pole-Aitken basin, shown in Fig. 3, presents the similarity with the major mascons on the nearside.

Disccusion and Conclusions: We have developed a new model objective function of inverse problem in spherical coordinate system. By inverting the gravity anomaly on the Moon, our method has the ability to directly estimate the local or global 3-D structures of density anomalies. If assuming the density of crust is 2.8 g/cm^3, the previous studies yielded a thickness of ~100 km on the farside. However, our 3-D density structures show that the value of negative anomalies is slightly smaller than 2.8 g/cm^3, and the location is at depth of 20-50 km, meaning that the thickness of farside highland may be less than 100 km.

Gravity inversion gives us geophysical supports to reveal the underground geological structures. Therefore, the satellite gravity data has potential value in reconstructing internal density distributions of planets.

Acknowledgement: This study is supported by the National Science Foundation of China (Grant No. 40774060).

References: [1] Wieczorek M. A. and Phillips R. J. (1998) *JGR, 103,* 1715–1724. [2] Wieczorek M. A. (2007) *Treatise on Geophysics*, 165–206. [3] Ishihara Y. S. (2009) *GRL, 36,* L19202. [4] Li Y. (1998) *Geophysics, 63,* 109–119. [5] Heck B. and Seitz K. (2007) *J. Geodesy, 81,* 121–136. [6] Backus G. E. and Gilbert J. F. (1967) *Geophys. J. R. Astr. Soc., 13,* 247–276. [7] Backus G. E. and Gilbert J. F. (1968) *Geophys. J. R. Astr. Soc., 16,* 169–205. [8] Liang Q. et al. (2009) *Sci. China Ser. G, 52,* 1867–1875. [9] Matsumoto K. et al. (2010) *JGR, 115,* E06007. [10] Ping J. et al. (2009) *Sci. China Ser. G, 52,* 1105–1114.

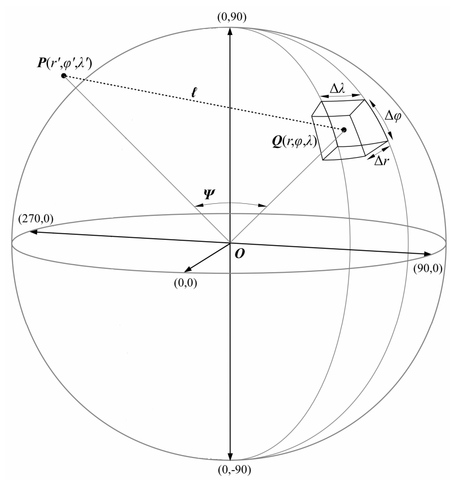

Figure 1. Geometry of tesseroid.

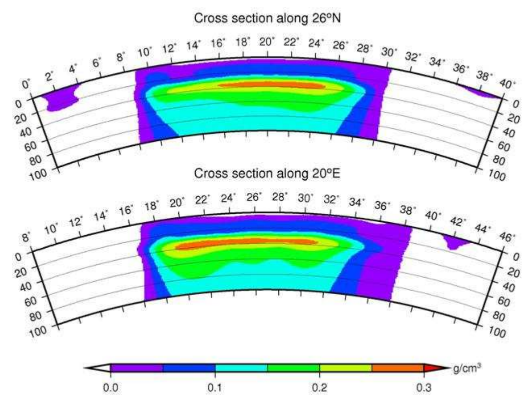

Figure 2. Cross sections of the 3-D density structure of the Mare Serenitatis along latitude of 26°N (upper) and longitude 20°E (lower).

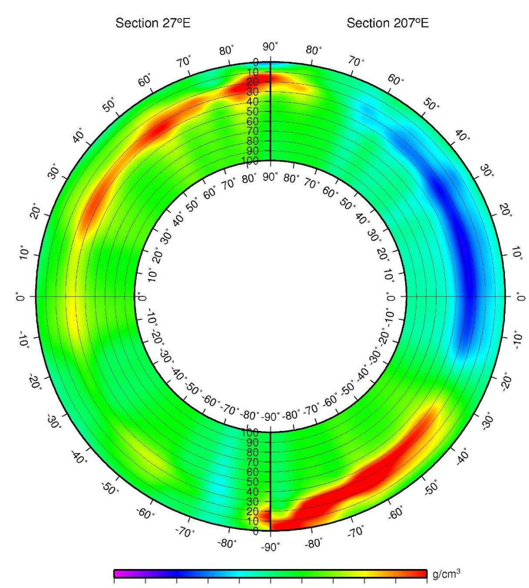

Figure 3. Cross section of global 3-D density structure of the Moon along longitude of 27°E and 207°E.