**Introduction:** To solve the self-gravitational interaction problem, recent releases of CTH [1,2] have included a parallel tree-based N-Body force solver [3] very similar to the approach used by SPH methods [4,5]. So equipped, CTH with its adaptive mesh refinement (AMR) ability [6] has been used to investigate the formation of the South-Pole-Aitken Basin [7] and candidate Moon forming impacts [8,9].

A well recognized **verification problem** for self gravity is the adiabatic collapse of an initially isothermal sphere. The test problem is that of an ideal gas (\(\gamma=5/3\)) sphere of mass \(M\) and radius \(R\) with a density distribution, \(\rho(r)=M/2\pi rR^2\). With an initially uniform internal specific energy of 5\% the characteristic gravitational energy (i.e. \(u=0.05GM/R\)) the sphere collapses under its own weight, producing shocks and rarefactions. Because of these features, it is a good problem for testing self gravity coupled to hydrodynamics. The problem has been extensively studied in one dimension using Lagrangian finite element methods with sufficient accuracy to be considered exact [10,11]. Results are typically expressed in dimensionless form normalized to characteristic density, \(\rho=3M/4\pi R^2\), time, \(t^*=(R^3/GM)^{1/3}\) and velocity, \(v^*=(GM/R)^{1/3}\). The test case shown in Fig. 1 uses an equal mass approximation refinement scheme that produces good agreement with the exact solution. With this approach, we mimic SPH methods by using successively lower resolution regions tied to density of 1/8th the next higher resolution to produce nearly equal mass zones (in AMR-CTH, each successive change in resolution is a factor of two in zone dimension and a factor of eight in volume).

**A candidate Moon forming impact** is shown in Fig. 2. The impact parameters are very close to run #24 from Canup and Asphaug [4], a large differentiated body with mass \(M_i\) obliquely impacts the proto-Earth at mutual escape velocity. The total mass \(M_T=1.02\) Earth masses, \(M_i/M_T=0.11\) and the scaled impact parameter[4,5] \(b'=L/L_{graz}=0.82\). Resolution of the calculation is tied to the equal mass approximation described earlier. The highest resolution region has 98 km cubical zones where density is greater than 10^{-2} g/cc. The simulation used 4-20 million zones during the calculation and ran for 6 days on 256 processors.

![Fig. 1](image1.png) **Fig. 1.** Adiabatic collapse of an initially isothermal spherical gas cloud. Comparison of the exact solution from Steinmetz and Müller [11] with AMR-CTH at \(t^*\times=0.77\) is shown.

![Fig. 2](image2.png) **Fig. 2.** High resolution CTH simulation of a candidate Moon forming impact. Time after impact (in hours) is shown for each panel.
The calculation shown in Fig. 2 used Melosh’s SiO₂ ANEOS description that includes diatomic and triatomic molecular clusters [12,13] for the mantle material. The core material of the impactor and proto-Earth were represented by standard ANEOS iron. Run #24 from Canup and Asphaug [4] used the Tillotson EOS for impactor and proto-Earth materials and produced 0.97 lunar masses orbiting beyond the Roche Limit. In comparison, this calculation, using ANEOS, produced 0.31 lunar masses in orbit beyond the Roche Limit. Canup [5] noticed a significant reduction in orbiting mass when ANEOS rather than the Tillotson EOS was used which may explain most of the difference we see.

Results of a numerical resolution study: Fig. 3 shows dependence on numerical resolution of several important measures of merit: predicted radius of the final Earth, predicted mass orbiting beyond the Roche Limit and predicted iron fraction in material orbiting beyond the Roche Limit. Since we continuously track these quantities, the plots show the predicted final outcome as a function of time during the course of each calculation. Several trends are apparent from the study. 1) It takes between 20-30 hours for the measures of merit to settle down to a steady state. 2) The time to achieve steady-state generally increases as zone size decreases. 3) While total predicted orbiting mass is less sensitive to numerical resolution than other parameters, predicted iron fraction is very sensitive to numerical resolution (note the variation of six orders of magnitude). 4) We have not achieved numerical convergence even for the highest resolution runs but the iron fraction trend suggests we may achieve adequate convergence at the next level of refinement (~50 km zones).

Conclusions: Differences in the numerical method, equations-of-state and initial conditions may account for many of the differences between this and other work. In collaboration with Canup and Barr from SwRI-Boulder, we have initiated studies to try to understand the nature of these differences. Here, we’ve presented results looking at the importance of numerical resolution. Future studies will look at material properties and initial conditions.


Fig. 3. Dependence of measures of merit using Canup & Asphaug criteria [4], as a function of numerical resolution. a) predicted Earth radius (Rₑ), b) predicted mass (in lunar masses) orbiting beyond the Roche Limit (Mₑorbit / MₑMoon) and c) predicted iron fraction in material orbiting beyond the Roche Limit (Mᵢron / MᵢRoche). Four simulations are shown with numerical resolutions of 781, 391, 195 and 98 km for the highest resolution region of each calculation.

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