# A SIZE DEPENDENT SCALING LAW BASED ON THE RATE DEPENDENT STRENGTH OF ROCKY BODIES.

J. Kimberley[1], K.T. Ramesh[1], O. S. Barnouin[2], C.M. Ernst[2]. [1]Dept. of Mechanical Engineering, Johns Hopkins University, Baltimore, MD 21218 (jamie.kimberley@jhu.edu, ramesh@jhu.edu). [2]Johns Hopkins University Applied Physics Laboratory, Laurel, MD 20723 (olivier.barnouin@jhuapl.edu, carolyn.ernst@jhuapl.edu)

**Introduction:** Scaling laws describing the effect of size on the catastrophic disruption of rocky bodies have been under development for the past 20 years. The predictions of these scaling laws have broad reaching implications related to our understanding of solar system evolution and the mitigation of threatening near-Earth objects. Historically, the specific energy that results in the largest remnant fragment of a collision being one-half the initial target mass, $Q*$, is used as the metric for investigating catastrophic disruptions. Numerous scaling laws [1, 2, 3, 4, 5, 6, 7, 8, 9] have been developed to describe the relation between target size and the catastrophic fragmentation threshold $Q*$, and are plotted in Figure 1 (after [10]). Although these scaling laws are based on different methods and assumptions, the plot breaks down into two regimes. For relatively large bodies, gravitational effects dominate the problem, and for relatively small bodies material strength dominates the disruption process. In the gravity regime there is a relatively steep increase in $Q*$ with body size. Note that there are two groups of lines in this regime, those corresponding to the specific energy required to shatter a body (which then may reaccumulate due to self gravity), and those corresponding to the specific energy required to shatter a body and disperse the fragments so that they do not reaccumulate. In the strength regime, the different scaling laws all converge to a common value for $Q*$ at body sizes of 1-10 cm (off to the left of the plot), because laboratory experiments at that size serve to anchor the scaling laws. The different slopes (generally on the order of -1/3 to -2/3 in a log–log plot) predicted by various researchers are the result of differing physics being incorporated into the scaling laws. For example, Holsapple [1] incorporated a strain rate dependent strength term based upon the linkup of a Weibull distribution of cracks in the target body, which lead to a relation of $Q* \propto R^{-1/3}$ for the assumed Weibull distribution, (here $R$ is the target radius). We present here a new scaling relation that results from a recently developed and validated micromechanics model for brittle solids based on the interaction and growth of a distribution of flaws in the target body.

**Rate Dependent Material Model:** A material model for the dynamic compressive strength of brittle solids has been developed based on the work of Paliwal and Ramesh [11], which explicitly incorporated the interaction of a distribution of preexisting flaws and crack growth dynamics. By identifying critical time and length scales involved in the problem, a universal relationship between the compressive strength of a brittle solid, $\sigma_f$, and the strain rate, $\dot{\varepsilon}$ has been demonstrated,

$$\sigma_f = \sigma_o\left(1 + \left(\dot{\varepsilon}/\dot{\varepsilon}_o\right)^{2/3}\right). \qquad (1)$$

Here $\sigma_o$ and $\dot{\varepsilon}_o$ are characteristic stress and strain rate based on the material and microstructural properties of the target [12]. This relation captures the rate insensitive response of geological materials at low rates as well as the sharp increase in compressive strength observed when compressed at high rates, and preliminary results show that it can be used to describe tensile failure as well. We incorporate this strength relation into a scaling law describing the variation of specific disruption energy as a function of target size in the strength regime.

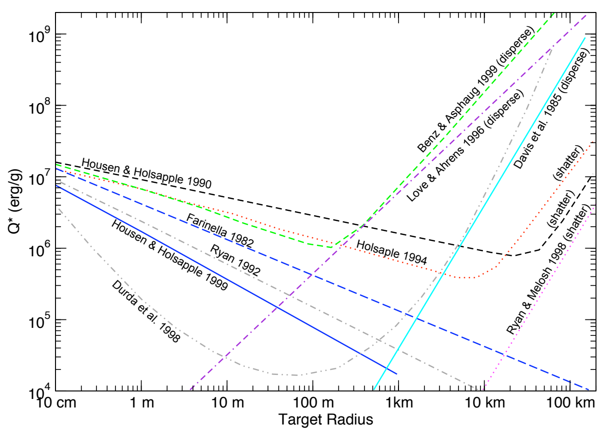


**Figure 1.** Thresholds for catastrophic disruption as a function of target size (after [10]).

**New Scaling Law in the Strength Regime:** In the strength regime Holsapple [1] has shown that

$$Q* \propto \left(\frac{S}{\rho}\right)^{3\mu/2} U^{2-3\mu}, \qquad (2)$$

where $S$ is a material property with units of stress describing the strength of the target, $\rho$ is the target density, $U$ is the impact velocity and $\mu$ is an exponent in the coupling parameter $mU^{3\mu}$ used in the point source approximation, and is typically taken to be 0.55 for rocky bodies [13]. By letting the strength measure in Equation (2) be equal to the compressive strength in

Equation (1), and approximating the strain rate in the body by $\dot{\varepsilon} = U/R$ the following scaling is arrived at,

$$Q^* \propto \left(\frac{\sigma_0}{\rho}\right)^{3\mu/2} U^{2-3\mu} + \left(\frac{\sigma_0}{\rho}\right)^{3\mu/2} \frac{1}{\dot{\varepsilon}_o^{\mu}} U^{2-2\mu} R^{-\mu}. \quad (3)$$

This equation is plotted for an impact velocity of 1 km/s in Figure 2 together with the data from laboratory experiments and an earlier scaling law described by [3]. Equation (3) predicts that the strength regime consists of two regions as illustrated by Figure 2. For small bodies there is a decrease in threshold specific energy with a slope of $-\mu$ in the log-log plot, chosen to be -0.55 here. This corresponds to the high strain rate regime of the strength–rate relationship (Equation (1)) and falls directly on top of the predictions of [3]. As the target size increases the average strain rate in the body decreases and the specific energy needed to disrupt a body reaches a plateau corresponding to the constant strength observed at low strain rates. Also shown in Figure 2 are lines for the shatter and disperse thresholds in the gravity regime given by [2]. The line for the dispersal threshold is approximated as being about two orders of magnitude greater than the specific energy required to shatter a body.

**Discussion:** Several key observations can be made from Figure 2. The first is that for a given impact velocity, bodies in the 10-1000m size range can be significantly harder to disrupt than would be predicted by [3]. Thus we may expect bodies in this size range to be monolithic shards. This scaling may also have implications for mitigating NEO's. Since bodies of this size range are harder to disrupt it may be possible to "bump" them to change their trajectory instead of only utilizing complicated gravity tractor scenarios currently accepted as the ideal method of mitigation.

Another observation is that there is a significant difference in the likelihood of generating rubble-pile bodies from a single impact event. Rubble-pile bodies result from impacts that shatter but do not disperse a target. This corresponds to areas above the thresholds in the strength regime, and between the shatter and disperse predictions in the gravity regime, as shown by the hatched regions in Figure 2. The scaling developed in [3] predicts that rubble-pile bodies are formed in the hatched regions, while the scaling developed here predicts formation only in the cross-hatched region. Our results show that only bodies larger than 10 km are likely to shatter and reaccumulate, whereas the scaling of [3] predicts that bodies as small as 1 km could be turned into rubble-pile bodies. We know that small rubble-pile bodies exist (such as Asteroid 25143 Itokawa). The new scaling presented here indicates that bodies of this type are likely fragments of a larger body that had similar post-impact trajectories, allowing for new small rubble-piles to accumulate. This notion is supported by the observation of large amounts of fine particles on Itokawa, which would likely have been produced by impact events that would fully disperse a body of this size range [14]. Thus it is likely that Itokawa is comprised of fragments of a larger body that accumulated some regolith from the parent body.

**References:**


[1] Holsapple, K. (1994) *Planetary And Space Science,* 42, 12, 1067-1078.
[2] Housen, K. R. and Holsapple, K. A. (1990) *Icarus,* 84, 1, 226-253.
[3] Housen, K. R. and Holsapple, K. A. (1999) *Icarus,* 142, 1, 21-33.
[4] Farinella, P. et al. (1982) *Icarus,* 52, 3, 409-433.
[5] Durda, D. D. et al. (1998) *Icarus,* 135, 2, 431-440.
[6] Benz, W. and Asphaug, E. (1999) *Icarus,* 142, 1, 5-20.
[7] Ryan, E. and Melosh, H. (1998) *Icarus,* 133, 1, 1-24.
[8] Davis, D. R. et al. (1985) *Icarus,* 62, 30-53.
[9] Love, S. G. and Ahrens, T. J. (1996) *Icarus,* 124, 141-155.
[10] Holsapple, K. et al. (2002) *Asteroids III,* 443-462.
[11] Paliwal, B. and Ramesh, K. T. (2008) *Journal Of The Mechanics And Physics Of Solids,* 56, 3, 896-923.
[12] Kimberley, J. and Ramesh, K. T. (2010) *LPS XLI*, Abstract #2543.
[13] Holsapple, K. A. (1993) *Annual Review Of Earth And Planetary Sciences,* 21, 333-373.
[14] Barnouin-Jha, O. S. et al. (2008) *Icarus,* 198, 1, 108-124.


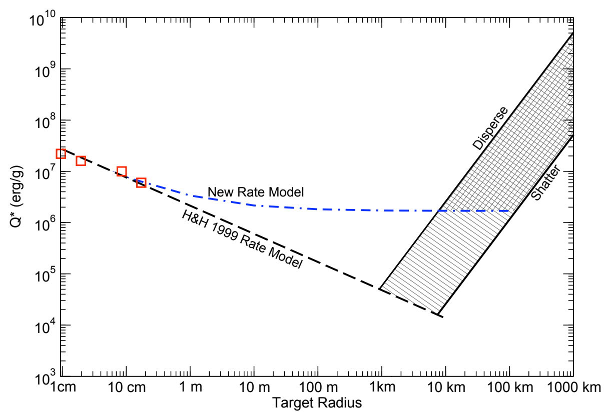


**Figure 2.** Comparison of new strength regime scaling with that of [3].