EVOLUTION OF SMALL, RAPIDLY ROTATING ASTEROIDS. D.J. Scheeres, *U. Colorado, Boulder (scheeres@colorado.edu)*, P. Sánchez, *U. Colorado, Boulder*.

We explore the evolution of small asteroids spun to rapid rotation rates by the YORP effect. If these asteroids are modeled as rubble piles with a characteristic grain size we find that van der Waals cohesion can stabilize a small asteroid against disruption even significantly beyond the surface fission limit discussed previously for asteroids. We trace out the possible evolution of such small bodies, noting that they may split repeatedly. Each split may cause the body to enter a complex rotation state, and will make the body more stable and able to be rotationally accelerated to even faster rotation rates at which it may fission again. A body experiencing such an uninterrupted sequence of fission events can be completely disassembled at minimum within a few YORP timescales of the initial body. We discuss this model in the context of asteroid 2008 TC3.

Simple Asteroid Model We analyze a simple model for a rubble pile asteroid, a cube with sides of length D containing $N_s \geq 2$ grains across each edge for a total of N_s^3 grains in the body, each of radius $r = D/(2N_s)$. The total mass of the object is ρD^3 where ρ is its density. We assume that each grain contacts 6 other grains (except at the edges). For analytic tractability we make the grains the same size and arranged in a cubic lattice which makes for a porosity of $\sim$50%, similar to what is seen in actual asteroids. We assume the configuration spins and has self-gravity between all of its different components. We also assume that granules in contact exert van der Waals cohesion between each other.

Internal Forces We first consider the forces internal to the body in a single direction perpendicular to the spin axis. Take a coordinate frame fixed in the body with the x axis perpendicular to one of the faces of the cube. Split the body perpendicular to the x-axis by a non-dimensional distance $0 \leq \mathcal{R} \leq 1$ so that the mass of one component is $M_1 = \rho D^3 \mathcal{R}$ and of the other $M_2 = \rho D^3(1-\mathcal{R})$. The distance between the centers of mass of these two components will always be a fixed value $D/2$ independent of $\mathcal{R}$.

The relative force between the two components will be the sum of the gravitational force (approximated as two masses centered at their centers of mass), d'Alembert inertial force, and cohesive forces at the interface between the two bodies. For our simple system these are

$$F_g = -4\mathcal{G}\rho^2 D^4 \mathcal{R}(1-\mathcal{R}) \quad (1)$$

$$F_s = \frac{1}{2}\rho D^4 \mathcal{R}(1-\mathcal{R})\Omega^2 \quad (2)$$

$$F_c = -\frac{1}{2}N_s^2 A r = -\frac{AD^2}{8r} \quad (3)$$

where $\mathcal{G}$ is the gravitational constant equal to 6.673×10^{-11} m^3/(kg s^2), Ω is the rotation rate of the body, and A is the Hamaker constant of the material divided by the contact spacing and equals $\sim$ 0.05 N/m for material analogous to lunar regolith [1,2], and r is the particle grain radius in meters. Thus the total force across an interior plane in the positive direction is

$$F_T = 4\mathcal{G}\rho^2 D^4 R(1-R)\left[\frac{\Omega^2}{\Omega_c^2} - 1\right] - \frac{1}{8r}AD^2 \quad (4)$$

where $\Omega_c^2 = 8\mathcal{G}\rho$. We see that if the cohesion is zero ($A = 0$), the body can separate for all $\mathcal{R}$ when $\Omega \geq \Omega_c$. However, once there is some cohesion the body no longer splits uniformly. Instead, we find that the total internal force will be positive when $(\Omega/\Omega_c)^2 \geq 1 + \frac{A}{64r\mathcal{G}\rho^2 D^2 R(1-R)}$ and see that the minimum spin rate for separation of the body occurs at $\mathcal{R} = 1/2$ when $\mathcal{R}(1-\mathcal{R})$ is maximized at the center of the body.

Internal Stress and Failure Criterion For our simple model, we can immediately identify the stress across different internal planes. We compute the stress across three planes, two that bisect the body perpendicular to the y and z axes, and one that cuts the body into two portions perpendicular to the x axis. The stresses in the axis directions are then all principle stresses and are computed by dividing the internal forces across the plane by the area, equal to D^2

$$\sigma_{xx} = 4\mathcal{G}\rho^2 D^2 \mathcal{R}(1-\mathcal{R})\left[\left(\frac{\Omega}{\Omega_c}\right)^2 - 1\right] - \frac{A}{8r} \quad (5)$$

$$\sigma_{yy} = \mathcal{G}\rho^2 D^2\left[\left(\frac{\Omega}{\Omega_c}\right)^2 - 1\right] - \frac{A}{8r} \quad (6)$$

$$\sigma_{zz} = -\mathcal{G}\rho^2 D^2 - \frac{A}{8r} \quad (7)$$

agreeing in general with [3], which were derived for an ellipsoidal body and did not have cohesion specifically incorporated into the stress. For the situations of interest to us, $\Omega > \Omega_c$, and thus we note that the stress σ_{xx} is maximized at the body center, $\mathcal{R} = 1/2$. Due to this in the following we only consider the stress on planes that cut through the center of mass of the body, as this will be the first location to fail in general.

Following Holsapple [3] we apply the Drucker-Prager failure criterion:

$$\frac{1}{\sqrt{6}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2} \leq k - 3sp$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the principal stresses, k is the cohesive shear stress for failure at zero pressure, $3p = \sigma_1 + \sigma_2 + \sigma_3$ and $s = 2\sin\phi/\left[\sqrt{3}(3-\sin\phi)\right]$, where ϕ is the friction angle and is taken to be 45° by Holsapple. For the failure shear stress we assume a simple model where $k = \mu|F_c|/D^2$, where $\mu \sim 1$ is a Coulomb friction coefficient. Evaluating the failure criterion we find

$$\frac{3s\mathcal{G}\rho^2}{\mu+3s}\left[\left(\frac{2}{3} + \frac{1}{3\sqrt{3}s}\right)\left(\frac{\Omega}{\Omega_c}\right)^2 - 1\right] \leq \frac{A}{8r} \quad (8)$$

Evaluating this inequality for an assumed bulk density of $\rho = 2 \times 10^3$ kg/m^3, a cohesion constant of $A = 0.05$ N/m, and a friction angle of $\phi = 45^\circ$ gives us a relationship between maximum grain size, body spin rate, and body size

$$r \quad \leq \quad \frac{21.928(1.068+\mu)}{\left[(1063\Omega)^2 - 1\right] D^2} \sim \frac{4 \times 10^{-5}}{(\Omega D)^2} \qquad (9)$$

Applying this to asteroid 2008 TC3 with a spin period of 100 seconds ($\Omega = 0.0628$) and size of $D \sim 4$ m [4], provides an upper limit of $r \leq 0.3 \times 10^{-3} (1 + \mu)$. Thus for $\mu \sim 1$ we see that millimeter-sized grains should have sufficient cohesive force to keep the body stable. This is consistent with the observed morphology of 2008 TC3 and motivates more detailed numerical analysis of this system using precise computations. We also note that away from the mid-plane of the asteroid, the total stresses will decrease and hence cohesion between larger-sized grains may be sufficient for stability.

Size and spin evolution Now assume we have a rubble pile asteroid characterized by a grain size r^*, meaning that it will fail when $\Omega D > \sqrt{4 \times 10^{-5}/r^*}$ and ideally fission into two pieces, failing along an internal plane of weakness either defined by fewer contacts between grains, an embedded larger component, or some other morphology that leads to weakness. Due to the rapid spin rate of the body, the two components will immediately escape each other. Furthermore, the two components will likely immediately enter a complex rotation state, as the moments of inertia of each separate body will not be naturally aligned with the principle axes, whereas the spin vector of the components will be conserved on either side of the splitting. Thus, we would expect the products of such a fission to be rapidly spinning bodies in complex rotation states, which again is consistent with known members of the asteroid population and with 2008 TC3 in particular. It is instructive to note that our ideal "cube" body will immediately enter into a pure long-axis rotation mode upon fission, again consistent with the long-axis mode rotation state of TC3 [4].

Assuming a uniform grain size r^* that controls the strength of internal cohesion, the new spin rate for failure of the fission products will be increased by a factor D/D', where D' is the new effective size of the body. If we assume that the body splits in half and arbitrarily model each portion as a cube again with a new effective size, D', then $D' \sim D/2^{1/3} = 0.8D$ and the body can spin up to $\Omega' = 2^{1/3}\Omega$, or approximately 25% faster, before it undergoes failure again.

Now assume that YORP spins the body up again. We take the YORP acceleration model from [5] which can be written as

$$\dot{\Omega} \quad = \quad P\frac{D\mathcal{C}_0}{2M} = \frac{P\mathcal{C}_0}{\rho}\frac{1}{D^2} \qquad (10)$$

where P is the average solar radiation pressure acting on the body, D is the size of the body, M its mass, and $\mathcal{C}_0$ is a dimensionless YORP characteristic value. Thus, if we assume a constant dimensionless YORP coefficient the rate of rotational acceleration increases as the body size decreases. Given an initial rotation rate Ω_0, we can compute the time it will take the body to be accelerated by a fraction f to $(1+f)\Omega_0$ as

$$T_f \quad = \quad \frac{\rho}{P\mathcal{C}_0} f \Omega_0 D^2 \qquad (11)$$

where we note that for small bodies (perhaps less than a few centimeters in size) the dependence transitions to being proportional to D [6].

If we assume that a fissioned body continues to rotationally accelerate to its next fission event, and that the size of the bodies decrease as estimated above, we can develop a minimum lifetime for a small body before it is completely dissipated by fission. Assume we start with a body of size D_0 and calculate the first time step as its YORP timescale, i.e. the time it takes to go from no rotation to a fission rate defined by Ω_0, or $T_o = \frac{\rho}{P\mathcal{C}}\Omega_0 D_0^2$. Then, after its first fission its new size will be $D_1 = D_0/2^{1/3}$ and its new fission rate will be $\Omega_1 = 2^{1/3}\Omega_0$. Then the time to accelerate from Ω_0 to Ω_1 given the new size D_1 is $T_1 = T_o(1 - 1/2^{1/3})/2^{1/3}$. By extrapolation we find that $T_2 = T_1/2^{1/3}$, etc., to find $T_n = T_0/2^{n/3}$, where we now define $T_0 = T_o(1 - 1/2^{1/3}) \sim 0.2T_o$, or about 20% of the YORP timescale of the body. Starting at time T_0, then, we find that the minimum lifetime of the body can be estimated as $T_L = T_0 + T_1 + \cdots T_n + \cdots$. Computing the summation $\sum_{i=0}^{\infty} \left(\frac{1}{2}^{1/3}\right)^i = \frac{1}{1-1/2^{1/3}}$, we then find that $T_L = T_o$, or that once the body starts to undergo its fission process the total lifetime of the body is approximately equal to its initial YORP timescale. If the body initially starts at a zero spin rate then it will take a minimum of ~ 2 YORP timescales for it to be completely dissipated by fission. We note that the YORP dependence on body size transitions to $1/D$ for small sizes [6]. This would increase the total lifetime, as then the time to spin up to fission becomes constant for subsequent products, proportional to $\Omega_n D_n$, and the sum of the individual lifetimes no longer converge. Still, after some finite number of fissions the resulting products will be equal to the smallest grains in the asteroid.

Again, let us consider 2008 TC3. Assuming that it has an initial non-dimensional YORP coefficient of 0.005, which is typical of the bodies evaluated in [5], that it lies in a circular orbit at 1 AU, has an initial rotation period of 100 seconds, and an initial mean size of ~ 4 m [4] we find the YORP timescale to be about 3000 years, which would be a lower-bound on the minimum lifetime of the object. Should an evolving rubble pile undergo a period of YORP deceleration and transition through a near-zero spin state the total lifetime of the object could be increased substantially.

References: [1]: Perko et al., Journal of Geotechnical and Geoenvironmental Engineering 127(4): 371-383 (2001). [2]: Scheeres et al., Icarus 210: 968-984 (2010). [3]: Holsapple, Icarus 187: 500-509 (2007). [4]: Scheirich et al., Meteoritics & Planetary Science 45(10-11): 1804-1811 (2010). [5]: Scheeres, Icarus 188: 430-450 (2007). [6]: Breiter et al., MNRAS 401: 1933-1949 (2010).