ROBUST ORBITAL REFINEMENT OF THE APOLLO TRAJECTORY DATA FOR THE AMES STEREO PIPELINE.

T. Kim[1], Z. M. Moratto[1], A. V. Nefian[1,2] , S. Ly[3], C. Demonceaux[4], and D. Fofi[4], [1]NASA Ames Research Center, Moffett Field, CA 94035, USA (taemin.kim@nasa.gov), [2]Carnegie Mellon University, USA, [3]University of Picardy, France, [4]University of Burgundy, France.

Introduction: Since 2007, the NASA Lunar Mapping and Modeling Project (LMMP) has been actively developing maps and tools to improve lunar exploration and mission planning. One of the requirements for LMMP is to construct geo-registered digital elevation models (DEMs) from historic imagery [1]. A joint effort between Arizona State University and NASA Johnson Space Center finished scanning the original film negatives [2]. The Intelligent Robotics Group at NASA Ames Research Center has developed the Ames Stereo Pipeline (ASP), a collection of cartographic and stereogrammetric tools for automatically producing DEMs from images acquired with the Apollo Metric Camera (AMC) during Apollo 15-17 [3].

The Bundle Adjustment (BA) in the ASP corrects the three-dimensional postures of cameras and the locations of the objects simultaneously to minimize the error between the estimated location of the objects and their actual location in the images. The BA requires good initial estimates of satellite positions within 1km error; however, the Apollo metric data contains unexpectedly large error in their radial measurements. Considering the two-body dynamics of the satellite and Moon, we can adjust the measurements to be consistent with their dynamics. The subsequent cartographic products of the ASP are improved by the accurate BA.

An energy-based method estimating the orbital trajectory of Apollo satellites is proposed to provide reliable initial estimates of satellite positions to the ASP. Camera position errors have a direct effect on the accuracy of DEMs produced by the ASP. The Apollo trajectory data, however, have large errors that throw off the ASP. This paper addresses the problem of refining the orbital trajectory consistent with the two-body dynamics of the satellite and the Moon, and integrating the refinement process into the BA of the ASP.

Robust Estimation of Two-body Dynamics: Suppose a satellite at $\mathbf{x}$ orbiting the Moon (**Figure 1**a). Regardless of the satellite mass, the governing equation of the satellite is described by a second-order ordinary differential equation:

$$\ddot{\mathbf{x}} = -\frac{GM\mathbf{x}}{\|\mathbf{x}\|^2}, \qquad (1)$$

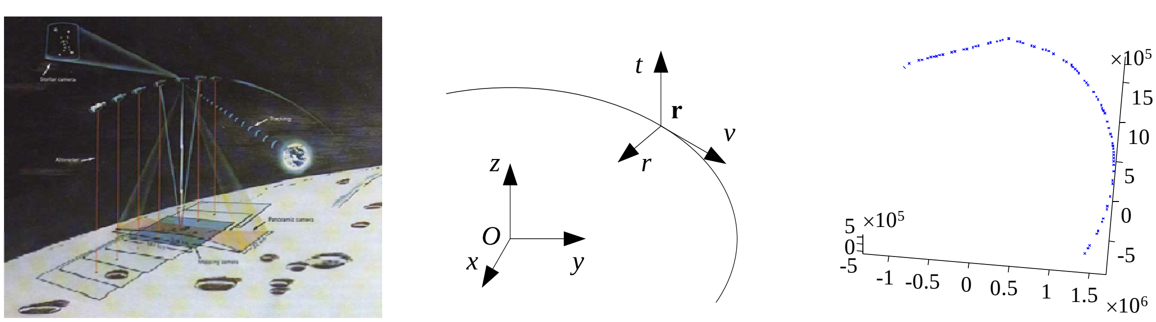
where G is the gravitational constant and M is the mass of the Moon. The error mainly occurs in the radial direction so that the Cartesian coordinate is decomposed into tangential coordinates $\mathbf{r}$ (**Figure 1**b). The goodness of fit for the radial direction is used to evaluate the data.

A robust estimation method is proposed to screen ouliers from observations by evaluating their goodness of fit [4]. Suppose a set of observations $P = \{x_k\}_{k=1}^{n}$ from a normal distribution with some outliers (**Figure 1**c). Their likelihood to be inliers is represented by a weight vector $\mathbf{w} = [w_k] \in [0,1]^n$. For their inlier portion, the weighted mean and sum of squared error (SSE) is defined by

$$\bar{x} = \frac{1}{w}\sum_{k=1}^{n} w_k x_k \text{ and } S^2 = \sum_{k=1}^{n} w_k S_k^2, \qquad (2)$$

where $w = \sum_{k=1}^{n} w_k$ is the total sum of weights and $S_k^2 = (x_k - \bar{x})^2$ is the kth squared error.

The weights of observations are updated by their



(a) The Mapping Cameras System (b) Tangential Decomposition (c) Apollo 17 Orbit 10

Figure 1: AMC Data System. (a) The AMC captures a series of pictures of the Lunar surface (Image Credit: NASA SP-362, Chapter 1). (b) The position of the satellite orbiting the Moon is decomposed into the tangential coordinates. (c) Satellite positions for the Apollo 17 Orbit 10 contain outliers in the first part of orbital trajectory.

goodness of fit to the normality. The individual and total mean squared error is defined by

$$s_k^2 = \frac{S_k^2}{\nu_k} \text{ and } s^2 = \frac{S^2}{\nu}, \qquad (3)$$

where ν_k and ν are the kth and total degrees of freedom such that

$$\nu_k = 1 - \frac{w_k}{w} \text{ and } \nu = w - \frac{1}{w}\sum_{k=1}^{n} w_k^2 . \qquad (4)$$

The statistic to test statistical significance for x_k is

$$r_k^2 = \frac{s_k^2}{s^2} \sim F_{\nu_k,\nu} . \qquad (5)$$

Through the hypothesis testing, the individual weight is updated by the gradient descent rule:

$$\Delta w_k = \eta\left\{1 - F_{\nu_k,\nu}(r_i^2) - \alpha\right\}, \qquad (6)$$

where α and η are significance level and learning rate.

Suppose we have n observations $P = \{\mathbf{z}_k\}_{k=1}^{n}$ in the tangential coordinates of the trajectory. Assumed $\mathbf{z}_k \sim N(\mathbf{r}_k, \mathbf{\Sigma})$, where $\mathbf{r}_k$ is the estimated satellite position to satisfy (1) and $\mathbf{\Sigma}$ is the covariance matrix along the trajectory, the log-likelihood function weighted observations is derived:

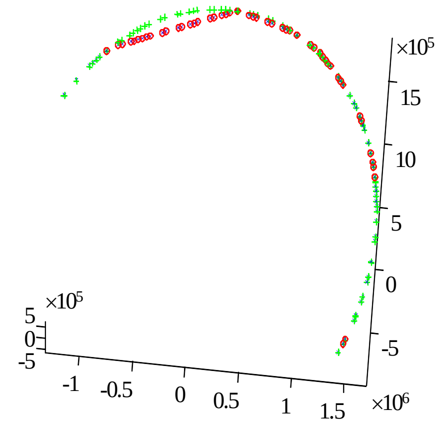
$$L = -2\ln 2\pi - \frac{1}{2}\ln|\mathbf{\Sigma}| - \frac{1}{2}\sum_{k=1}^{n}\frac{w_k}{w}e_k^2 \qquad (7)$$

where $e_k^2 = (\mathbf{z}_k - \mathbf{r}_k)^T \mathbf{\Sigma}^{-1}(\mathbf{z}_k - \mathbf{r}_k)$ is the squared Mahalonobis distance.

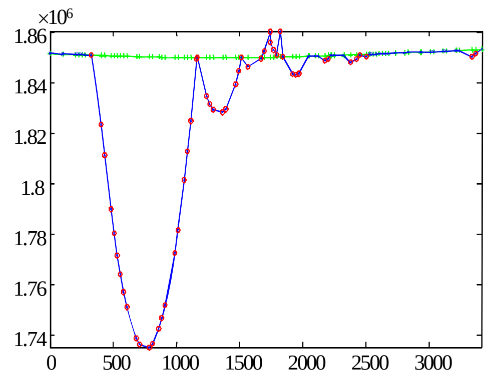
Experimental Results: The Apollo 17 trajectory data is available with the scanned images [2]. The significance level and learning rate are all set to 0.01 by extensive experiments. All weights are uniformly initialized to 0.5 and updated by (6) to the convergence. The blue line in **Figure 2** shows the large error in the first half of the Apollo 17 Orbit 10 trajectory. In particular, the radius of the Moon is known to be around 1737.4km so that the satellite almost hits the Moon with the Apollo 17 trajectory data as shown in the radial trajectory (**Figure 2**b).

The outliers are effectively screened by the proposed method so that the smooth trajectory is recovered only from the inliers. The green + in **Figure 2** indicates the new estimate of the trajectory which is crossed in blue. The outliers having the final weights less than 0.5 are circled in red. However, the final weights become either 0 or 1 at the convergence even though they are not forced to be Boolean.

Conclusion: A robust estimation method is proposed to exclude outliers in the Apollo trajectory data. For all observations in the trajectory, their goodness of fit is evaluated by the statistical testing and then their weights initialized uniformly to 0.5 are updated by the gradient descent rule. With updated weights, the trajectory is obtained by weighted maximum likelihood estimation. The orbital trajectory is recovered to be consistent with the two-body dynamics of a satellite and the Moon and to integrate the refinement process into BA of the ASP.



(a) Apollo 17 Orbit 10



(b) Radial Trajectory

Figure 2: Orbital Refinement (blue line with x for observations, red circle for outliers, and green + for new estimates). (a) Satellite station positions for the Apollo 17 Orbit 10 are refined to screen outliers. (b) The radial trajectory shows the outliers clearly.

References: [1] A. V. Nefian et al. (2010) LPS 41, Abstracts #1555. [2] S. J. Lawrence et al. (2008) LPI Contributions 1415, Abstract #2066. [3] Z. M. Moratto et al. (2010) LPS 41, Abstracts #2364. [4] T. Kim et al. (2010) LNCS 6454, 283-291.

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