ON A MODEL OF SPOKES ORIGIN IN THE CELESTIAL MECHANICAL SYSTEMS. N. I. Perov, Astronomical Observatory, State Pedagogical University, Respublikanskaya St, 108. Yaroslavl, 150000. Russia. E-mail: n.perov@yspu.yar.ru

Introduction: In the previous paper [1] we considered the model of the giant planets rings arcs motion in the frame of the restricted 4-body problem. Below a model of the rings spokes origin, based on the 10-body problem, is under consideration.

In accordance with work [2] the spokes are intermittently appearing radial marking in Saturn's B ring that are believed to form when micrometer – sized dust particles are levitated above the rings by electrostatic forces. First observed by the Voyagers, the spokes disappeared from October 1998 until September 2005, when the Cassini spacecraft saw them reappear. The trajectories of the charged dust particles comprising the spokes depend critically on the background plasma density above the rings, which is a function of the solar elevation angle. This notable effect is capable of stopping spoke formation entirely and restricting the size of the particles in the spokes [2]. These features, which are composed of micrometer –sized dust particles, are typically 10,000 km in length and 2000 km in width [2].

Celestial Mechanical Model of Spokes Motion: Let's consider a *central configuration* of 10 bodies – planet of mass m_1 and eight satellites of mass m_i, located in vertexes of an nonagon with the sides of a_i , $m_1>m_i$, i=2,…,8, and particle with zero mass of m_0 , in initial moment of time placed in a straight line, connecting m_1 and the middle point, placed between m_5 and m_6 (Fig.1). The bodies with mass m_1, m_2, …, m_0 rotate uniformly around the axes passed through the system center of mass C [3], [4].

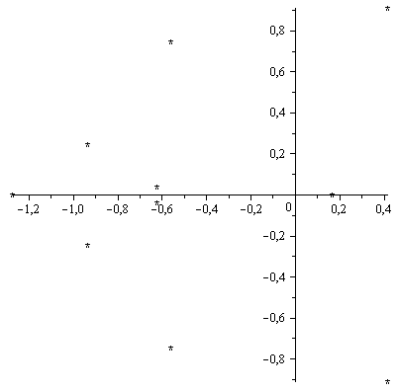

Fig. 1. In the ten-body system - a planet m_1, eight satellites with mass m_i, (i=2, …, 8), a particle m_0, with negligible mass, - the ring's spoke for $m_i/m_1>10^{-1}$ is formed (in initial moment of time m_0 coincides with a point of libration L – is extremely left point).

We denote $\boldsymbol{R}_i$ are the radii-vectors of the satellites.
The differential equations of the major body's motion may be represented in the form

$$d^2\boldsymbol{R}_i/dt^2=-\sum_{j=1}^{j=9}\{Gm_j(\boldsymbol{R}_i-\boldsymbol{R}_j)/|\boldsymbol{R}_i-\boldsymbol{R}_j|^3\},$$
$$(i,\ j=1,\ldots,9,\ i\neq j). \quad (1)$$

Here G is the gravitational constant.
From the determination of the central configuration [5] it follows

$$d^2\boldsymbol{R}_i/dt^2=-\omega^2\boldsymbol{R}_i,\ \ i=1,\ldots,9, \quad (2)$$

where ω is an angular velocity of the dynamical system.
For the central configuration

$$d^2\boldsymbol{R}_i/dt^2\cdot\boldsymbol{R}_{i\perp}=0,\qquad i=1,\ldots,9. \quad (3)$$

Here $\boldsymbol{R}_{i\perp}$ is the vector directed perpendicular to the vector $\boldsymbol{R}_i$.
Moreover, in the system coordinates refer to the center of mass we have

$$\sum_{i=1}(m_i\boldsymbol{R}_i)=0,\ i=1,\ 9. \quad (4)$$

Assume $m_2=m_9$, $m_3=m_8$, $m_4=m_7$, $m_5=m_6$.

Example: Let's put m_1=1, $m_2=m_9$=0.3, $m_3=m_8$=0.2, $m_4=m_7$=0.1 and using unit of length R_2=1and unit of time for which G=1 find the positions of major bodies and points of libration from the equation (1) – (4). So R_1=0.16461908592636, R_3=0.933971843517916, R_4=0.96833637762843, R_5=0.625442148891054, m_5=0.00038451299383, r_{12}=0.943768923158365, r_{13}=1.04158006296209, r_{14}=1.128364086469174, r_{15}=0.78982250506548, r_{23}=0.991531813503350, r_{34}=0.62390241377020, r_{45}=0.374994682580414, r_{56}=0.075674638877066.
r_{ij} is the distances between neighbors bodies.

Coordinates of the libration points L are determined with help of equations similar the equations of (1) and (2) are equal to x_L=-1.275708180196753845, y_L=0.

There are 4 additional libration points in this celestial mechanical system L_2'', L_3'', L_4'', L_5''. Abscesses of these points are equal to:
X_{L2}=-0.941854238089; X_{L3}=-0.654754928787;
X_{L4}=-0.621825324096; X_{L4}=-0.599365726912,

and theirs ordinates equal 0.

For the point of libration L′ which is extremely right point from the center of the mass we have R_L=1.103549732941294129.

Lyapnuv's theorems and numerical integration show the considered dynamical system of 10 bodies is unstable in the plane *Oxy*. (Fig.2 and Fig.3).

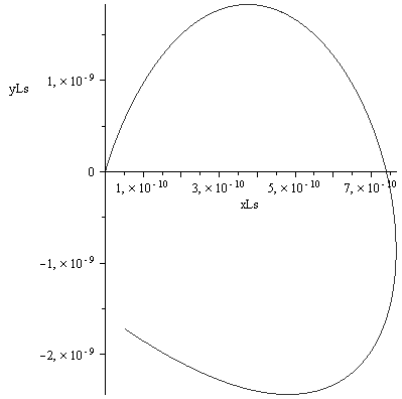

Fig.2. Motion of the body with zero mass around the point of libration L′ for x=*y*=0, *dx/dt*=*dy/dt*=0. Revolution number of the major bodies equal 41.75.

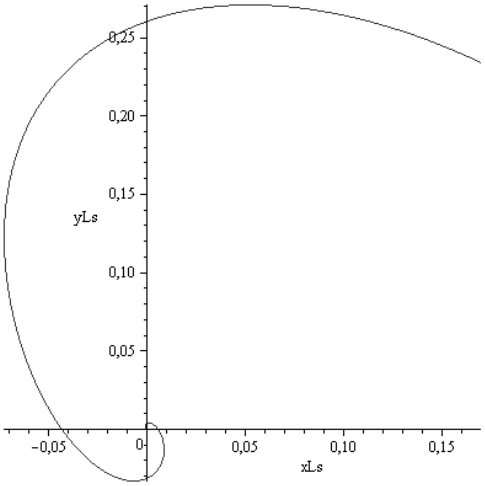

Fig.3. Unstable motion of the body with zero mass around libration point L′ and "regular trajectory" for x=*y*=0, *dx/dt*=*dy/dt*= 0.00000001. Revolution number of the major bodies equal 4.75.

It should be noted for the small mass $m_i/m_1 < 10^{-4}$ major bodies form an arc of a ring and the extremely left point of libration *L* is stable in the plane *Oxy*, but there are only two points of libration in this dynamical system - the extremely left and the extremely right points of libration [4]. (Fig.4).

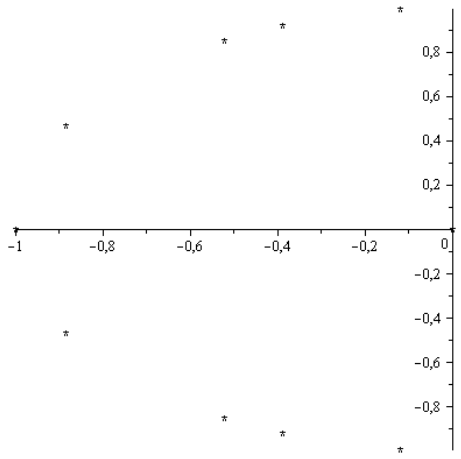

Fig.4 10 - body gravitating system in the form of an arc of a ring with stable of the extremely left point of libration ($m_i/m_1<10^{-4}$).

Conclusion: In the celestial mechanical system of 9 major massive bodies, which form central configuration, for $m_i/m_1>10^{-1}$, the bodies of mass m_1, m_5 and m_6 and 6 points of libration form a spoke's structure (Fig.1).

In the celestial mechanical system of 9 major bodies with negligible mass, which form central configuration, for $m_i/m_1<10^{-4}$, the bodies with mass m_1 and 2 points of libration in fact do not form a spoke's structure (Fig.4).

In the next paper we shall consider the model of spokes origin based on the spatial central configuration [6].

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References: [1] Perov N. I. and Kondratieva A. V. (2011) *LPS XLII*, Abstract # 1608. [2] Mitchell C. J. et al. (2006) *Science.*, *312*, 1586-1589. [3] Perov N. I. (2010) *Minor Bodies: Migration and Searching.* Lambert. Saarbrucken. Germany. 300 pp. (In Russian). [4] Perov N. I. et.al. (2011) *Theoretical methods of localization in the space-time of undiscovered celestial bodies.*YSPU. Yaroslavl. 204 pp. (In Russian). [5] Winter A. (1941) *The Analytical Foundations of Celestial Mechanics.* Princeton. New York. [6] Hampton, M. and Jensen, A. (2011.) *CM, 109*, 321-332.