THE PROBLEM OF THE GRAVITATIONAL ACCRETION OF PARTICLES. T. R. Abdulmyanov, Kazan State Power Engineering University, Kazan, Russia (abdulmyanov.tagir@yandex.ru).

Introduction: The fundamental laws of gravitational interaction of bodies in classical celestial mechanics are Newton's laws. According to the laws the accelerating each of the two interacting bodies with masses M and m is $G(M+m)/r^2$. The main reason of the lack of details of accretion mechanism, the following. It is difficult to determine the relationship between the initial conditions for the motion of original particles and of the new particle that will arise as a result of accretion. If the initial conditions of motion is defined to one of the particle and to the second particle, would be their sum equal to the the initial condition of the new particle arosen due to accretion? In order to find the solution of the problem, it is need to consider more general equations of motion, based on the classic law of gravitation. The orbit of a particle or celestial body in this general model of motion can be an ellipse with a rotating apsidal line.

The theory of revolving orbits of the celestial bodies in a central gravitational field have a rich history. Newton begun to study this problem in his classical work "Philosophiae Naturalis Principia Mathematica". Attempts to apply different models of motion with rotating apsidal line to the theory of the Moon there are in the works [4, 8]. In [3] a full historical analysis of various classical approaches to apsidal line rotating problem is given. Current status of mathematical aspects of the problem is seen in the works [5 – 7].

In this paper we attempt to detail the process of accretion using the more general equations of the motion with the law of egual areas for revolwing orbits. It's shown that a particle will have a revolving orbit as a result the gravitational-wave action generated by collision of two particles.

Revolving orbits in Newtonian model: Let us consider the motion of two particles m_1 and m_2 around a central body M. Suppose that the particle masses m_1 and m_2 are so small that these particles interact only with the central body M. Gravitational interaction between particles m_1 and m_2 is assumed to be missing. According to the Newton law each particle will have acceleration described by the following equations:

$$\frac{d^2\vec{r}_1}{dt^2} = \frac{G(M+m_1)}{r_1^2}\frac{\vec{r}_1}{r_1}, \quad \frac{d^2\vec{r}_2}{dt^2} = \frac{G(M+m_2)}{r_2^2}\frac{\vec{r}_2}{r_2}. \quad (1)$$

When two particles form a new particle due to accretion then the new particle will retain the properties of motion of the first and second particle. Newton proposed to extrapolate the second Kepler law into the case of orbits with a rotating line of apsides. Let's look the consequences, which Newton has received on the basis of his assumption. Suppose that the movement of celestial body mass m is so far from the main body mass M that the size and shape of the bodies m and M can be neglected. Suppose also that the gravitational force $F_1(r)$ body mass m will move in elliptical orbit, and as a result of the gravitational force $F_2(r)$ body mass m will move to an orbit with apsidal line rotating ellipse. Then for those two cases of motions in a polar coordinate system will apply the following equations of motion:

$$\frac{d^2r}{dt^2} + r\left(\frac{dv}{dt}\right)^2 = F_1(r), \quad \frac{d^2r}{dt^2} + r\left(\frac{du}{dt}\right)^2 = F_2(r), \quad (2)$$

where r is radius-vector of the bodies, v and u - polar angles of the elliptical orbit and of the orbit with rotating apsidal line respectively. According to Newton it is supposed that the motion of the bodies satisfy of the two following laws of egual areas:

$$r^2\left(\frac{dv}{dt}\right) = h_1, \quad r^2\left(\frac{du}{dt}\right) = h_2, \quad (3)$$

where h_1, h_2 - arbitrary constants. Using these two laws from the equations (2) we get:

$$F_2(r) - F_1(r) = \frac{h_2^2 - h_1^2}{r^3}.$$

Therefore, if all the above assumptions, the difference between $F_1(r)$ and $F_2(r)$ will follows - $(h_2^2 - h_1^2)/r^3$. This conclusion is the main conclusion by Newton, which can be seen from its proven theorems. Consequently, one of the first generalizations of the two-body problem, in which the celestial body moves on the rotating elliptical orbit, was considered by Newton. Other generalizations of the two-body problem can be fined in the papers [5 – 7]. All of these generalizations of the two-body problem are very interesting as the mathematical problems. Physical aspects of these generalizations of the two-body problem have so far not been found.

The model of accretion: Consider the version of the generalizations of the two-body problem, wich does not require changing of the main sense of the classical gravitational law. Suppose that the functions $F_1(r)$ and $F_2(r)$ from equations (2) are defined as follows:

$$F_1(r) = -\mu_1/r^2 = -\mu_1 s,^2 \quad F_2(r) = -\mu_2/r^2 = -\mu_2 s^2,$$

where $\mu_1 = G(M+m_1)$, $\mu_2 = G(M+m_1+m_2)$ are constants and $r = 1/s$. It is need to determine how will change the orbits of particles as a result of accretion.

Equations (2), taking into account the laws (3), can be written in the following form:

$$h_1^2 s^2 \frac{d^2 s}{dv^2} + h_1^2 s^3 = \mu_1 s^2, \; h_2^2 s^2 \frac{d^2 s}{du^2} + h_2^2 s^3 = \mu_2 s^2. \tag{4}$$

From (4) we get the following equations:

$$\frac{d^2 s}{dv^2} + s = \mu_1 / h_1^2, \; \frac{d^2 s}{du^2} + s = \mu_2 / h_2^2. \tag{5}$$

If the right parts of these equations are the same, they will have also same solutions. If the right parts are different, they solutions will be also different. Suppose that the right-hand sides of equations (5) are different and the functions s_1, s_2 are the solutions to these equations, respectively. In this case the function $\bar{s} = s_1 - s_2$ is the solution of the following equation in partial derivatives [1,2]:

$$\frac{\partial^2 \bar{s}}{\partial v^2} + \frac{\partial^2 \bar{s}}{\partial u^2} + \bar{s} = \mu, \tag{6}$$

where $\mu = \mu_1 / h_1^2 - \mu_2 / h_2^2$. Using the solution $\bar{s}$ of the equation (6), the solution s_2 can be presented as the difference $s_1 - \bar{s}$. Consequently, the orbit s_2 is formed due to the accretion of the particles m_1 and m_2, and as a result of gravitation-wave action $\bar{s}$ to the orbit s_1. In the case $u = kv$ the second of the equations (5) transforms to the following form and will have the following solution:

$$\frac{d^2 s}{dv^2} + k^2 s = k^2 \mu_2 / h_2^2, \tag{7}$$

$$s = (1 + e\cos kv)/p + \mu_2 / h_2^2,$$

where where k is an arbitrary constant. From equation (7) it follows that in the case $k^2 = 1$ and $m_2 = 0$ we obtain $s_2 = s_1$. That is, the orbit s_1, in this case will remain unchaged. If $k^2 \neq 1$ and $m_2 \neq 0$, then by the continuity of the solutions of the equations (5), the right-hand sides of the equation (5) should be proportional to each other, that is,

$$k^2 \mu_2 / h_2^2 = \mu_1 / h_1^2. \tag{8}$$

From the equation (8) we can obtain constant k by following formula:

$$1/k^2 = h_1^2 / h_2^2 [1 + m_2 /(M + m_1)]. \tag{9}$$

By equation (9), the followig cases are take place:

1) The solutions of the equations (5) are identical ($m_2 = 0$, $h_1 = h_2$).
2) In this case, the orbit s_2 is a rotating ellipse ($m_2 = 0$, $h_1 \neq h_2$).
3) This case corresponds to the accretion of two particles m_1 and m_2. As the result of accretion can be formed a new particle which have the Kepler's orbit ($m_2 \neq 0$, $k = 1$).
4) This case corresponds to the accretion of two particles m_1 and m_2. As the result of accretion can be formed a new particle which have the revolving orbit ($m_2 \neq 0$, $k \neq 1$).

Thus, the Kepler's orbit can be formed as a result of accretion of two particles m_1 and m_2. The elliptical orbits with a rotating line of apsides can be formed also as a result of accretion.

Conclusions: Phase formation of matter in the Solar, in which the gravitational accretion of particles could play a key role, little studied at present. It is possible that Saturn's rings in the active phase of the accretion of particles are at the present time. In this paper it is considered the model of accretion of the particles. It is shown that:

1) The accretion of the particles can be considered as a result of the gravitational-wave action, which is generated by collisions of these particles.

2) As a result of accretion of the particles may be formed a new celestial bodies with the Keplerian orbit, as well as the orbits of the body with a rotating line of apsides.

3) From this it follows, that the physical aspect of accretion of particles becomes more clear.

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