INVERSE PROBLEM OF THE LUNAR PHYSICAL LIBRATION BY OBSERVING STARS FROM THE LUNAR SURFACE. N. Petrova[1,2], T. Abdulmyanov[2] and H. Hanada[3], [1]Kazan Federal University, 18, Kremljevskaja str., Kazan, 420008, Russia, e-mail: nk_petrova@mail.ru; [2] Kazan State Power Engineering University, 51, Krasnoselskaja str., Kazan, 420066, Russia, e-mail: abdulmyanov.tagir@yandex.ru; [3] National Astronomical Observatory, 2-12 Hoshigaoka, Mizusawa, Iwate 023-0861, Japan, e-mail: hanada@miz.nao.ac.jp

Introduction: Rotation of celestial bodies is a kind of a key to the internal structure of a celestial body. In this connection, the lunar experiments aimed at the study of the Lunar Physical Libration (LPhL) are of great interest. Optical astrometric observations are one of the new trends for observations of the lunar rotation in a future lunar mission after the success of SELENE (Kaguya). One of the necessary stages of preparation for the upcoming experiments is the theoretical simulation of the future observations. In this report we present several results of the simulation of polar stars observation in the framework of the ILOM (In-situ Lunar Orientation Measurement) project [1].

The system of selenographic coordinates: In the framework of the current study we simulate the observation with an "ideal telescope" [2]: the telescope will be posed exactly at the lunar dynamical pole (the axis of its tube coincides with the principal inertia axis *C* of the Moon) and the axes of the CCD-array situated in the lens of the telescope will be ideally directed along the other two principal axes of inertia A and B. The motion of stars will be displayed relatively to the axes of inertia, which are rigidly connected with the lunar body. Reduction of rectangular ecliptical coordinates $\vec{E}$ of any star to the selenographycal coordinates $\vec{S}(t) = (x_s, y_s, z_s)^T$ may be done with the lunar libration angles $\tau(t), \rho(t), \sigma(t)$ on the basis of equation system, whose common expression can be written in the following form [2]:

$$\vec{S}(t) = \Pi(\tau(t), \rho(t), \sigma(t))\vec{E} \qquad (1)$$

Here Π – is a function formed by production of rotation matrixes, used for the transition from the ecliptic coordinate system to selenographic system.

Formulation of the problem: Under the *inverse problem* of LPhL we understand finding the values of libration angles $\vec{X}^o = \tau^o(t), \rho^o(t), \sigma^o(t)$ from the system of equations (1) by using selenographic coordinates of stars x_s^o, y_s^o, z_s^o measured during the observations.

In the inverse problem the angles of libration are considered as unknown variables described by the vector $\vec{X}(t) = (\tau(t), \rho(t), \sigma(t))^{\mathrm{T}} = (x_1(t), x_2(t), x_3(t))^{\mathrm{T}}$. Then the system of equation (1) can be rewritten:

$$\mathbf{F}(\vec{X}) = \prod_Z \left(F + x_1 - x_3 + 180^O\right) \times \prod_{\bar{X}} \left(-(I + x_2)\right) \times \times \prod_{\bar{Z}} (\eta + x_3) \times \vec{E} - \vec{S}^0 = 0. \qquad (2)$$

F is the distance of the mean longitude of the Moon from the mean longitude of its ascending (northward-bound) node η = ☊. The constant $I \sim 1^o 32.5'$ is mean inclination of lunar pole to ecliptic pole.

Jacobian of the system (2) turned out to be close to zero. Therefore the system is ill-posed. The systems with the Jacobian close to zero can be solved using the *gradient method* [3]. Since the inverse Jacobian matrix is not used in the realization of this method, it provides a good convergence for our type of functions $\mathbf{F}(\vec{X})$ within the given accuracy.

Analysis of the inverse problem solution: Solution of system (1) for stellar coordinates $\vec{S}(t) = (x, y, z)^T$ is obtained every 12 hours using the analytical theory [4, 5]. This is a *direct problem* of LPhL. As a result, we obtain a set of numerical values for selenographic coordinates of a star for a given observation period. Simultaneously we also obtain a set of values for physical libration angles $\vec{X}^c(t) = [\tau^c(t). \rho^c(t), \sigma^c(t)]^T$ for the whole observation period.

By solving the inverse problem the coordinates $\vec{S}(t)$ obtained at the previous stage are introduced into Eqs. (2). The system is solved using the gradient method with a given accuracy ε relative to the vector $\vec{X}(t)$. The values of $\vec{X}^c(t)$ calculated of the stage of the direct problem are taken as initial values. At the output we obtain series of values for $\vec{X}^o(t) = \left[\tau^o(t), \rho^o(t), I\sigma^o(t)\right]^T$ and compare them with $\vec{X}^c(t)$. The results of the comparison shows that the gradient method does not introduce significant errors into the values of $\vec{X}(t)$: actually, $\vec{X}^o(t) - \vec{X}^c(t) << \varepsilon$ for the whole time interval.

Estimation of influence of an inaccuracy in measuring the coordinates on the accuracy of libration angles: The ILOM project telescope will yield rectangular coordinates $x_s(t), y_s(t)$ for each star appearing in its field of view. For our analysis we have taken a fictitious star with coordinates equal to those of the mean Northern Pole (P_N) of the Moon at the initial moment of observation:

$$\lambda_{P_N} = \Omega + 90^o; \ \beta_{P_N} = 90^o - I$$

Using a random number generator we simulated different levels of random errors Δx, Δy for each instant of the measurement. The procedure of testing the sensitivity of libration angles to the measurement error was carried out according th the algorithm described in previous section. The difference is that before introducing the vector $\vec{S}(t)$ into (2) its components are made noisy by hand. The spectra $\vec{X}^o(t) - \vec{X}^c(t)$ were obtained for different values ε. A part of these spectra corresponding to 1 month and $\varepsilon = 10$ mas is depicted on Fig. 1. demonstrating the influence of measurement error Δx, Δy on $\Delta\tau$, $\Delta\rho$ and $I\Delta\sigma$.

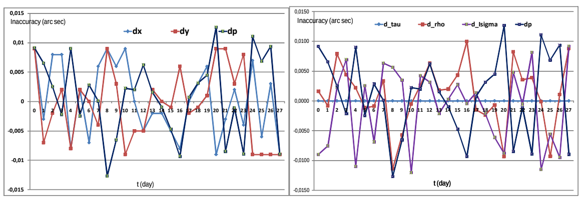

Fig. 1 The random inaccuracies in Δx, Δy, Δp (left) and the corresponding inaccuracies in the angles of libration (right).

Analyzing these diagrams we can make the following conclusions:

1. Throughout the whole observation period inaccuracies in libration angles $|\Delta\rho|$ and $|\Delta I\sigma|$ are of the same order as the inaccuracies in the polar distance $|\Delta \text{p}|$. More accurately we estimated that, if the inaccuracy in the determination of coordinates $\varepsilon = 1$ mas is achieved technologically, then the inaccuracy in the determination of LPhL angles will be less than $\sqrt{2}\varepsilon \leq$ 1.41 mas, i.e. $|\Delta\rho| \leq \sqrt{2}\varepsilon$ and $|\Delta I\sigma| \leq \sqrt{2}\varepsilon$.

2. At the same time, the value of $\tau(t)$ is independent on variation in x, y and, consequently, cannot be determined from the polar stars.

We can understand this phenomenon from geometry of physical libration: longitudinal librations depend on selenographic latitude δ proportionally to $\cos\delta$, which for the polar zone is close to zero.

Manifestations of the deformability of the lunar body in polar libration: The expansion of Petrova's analytical theory in the case of a deformable Moon was made on the basis of complements, calculated by Chapront et al. [6] to the Moons' libration theory [7] concerning tidal effects. We have carried out the comparison of data (coordinates $\vec{S}(t)$ of the fictitious pole and libration angles $\vec{X}$) obtained in the framework of LPhL theory for the rigid Moon and for the deformable Moon respectively ($k_2 = 0.02992$). At the stage of direct problem we calculated coordinates and libration angles for both models. At the next stage (inverse problem) we substituted the coordinates $\vec{S}^d$ obtained within the deformable Moon model into Eq. (2) considering them as observable data and solved this equation for unknown libration angles $\vec{X}$, the initial values for them $\vec{X}$ being taken from the rigid Moon model $\vec{X}^{rigid}$.

The residuals $\vec{X}^d - \vec{X}^{rigid}$ shown on Fig. 2 point to well-marked both periodical variations and constant shift ($\Delta\tau$ is insensitive to the changing of model).

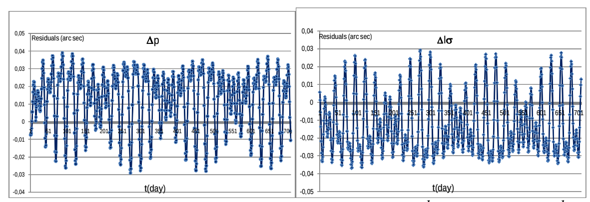

Fig. 2 Residuals in libration angles $\Delta\rho = \rho^d - \rho^r$, $\Delta\sigma = I(\sigma^d - \sigma^r)$ during 26 sidereal months.

We carry out the Fast Fourier Transform on the residuals. Obtained spectra for libration angles are shown on Fig. 3. At the same tame we calculate $\Delta\rho$ and $\Delta\sigma$ in analytical form. Strong harmonic $2F$ with the period of 13.62 days is very useful for the analysis. Weak component with the period of 9.077days corresponds to the (l-$2F$)-term. It may be also interesting for analysis, although its amplitude is on the verge of accuracy. We believe theat the constant small shifts in libration angles (-0,0117″ in $\Delta\rho$ and -0.2619″ in $\Delta\sigma$), may be also decreased by improving of k_2.

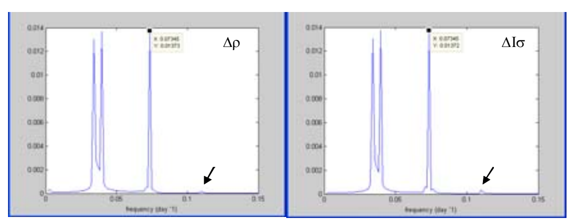

Fig. 3 Spectrum of residuals of Δρ(t) and ΔIσ(t). The unity of ordinate is arc sec.

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