POSSIBLE EFFECTS OF COSMOLOGICAL EVOLUTION ON THE ORIGIN OF THE SOLAR SYSTEM.
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Introduction: The possibility that there may be cosmological effects in planetary science has inspired our recent studies into what we have identified as Hubble tidal forces acting on the origin of the solar system [1-2], with related work on the Pioneer anomaly [3-4]. We will determine here the Hubble tidal effect explicitly in terms of the accepted Friedmann-Lemaitre (FL) accelerating universe (Concordance model) [5,6] in cosmology. Then the associated Binet-Clairaut equations will be derived. Finally, we will relate these results to studies of protoplanetary disk models for the origin of the Solar System.

From the fact that the onset of the accelerating universe occurs shortly before the origin of the Solar System, the possibility that the Hubble tidal pertubation might have triggered this arises [1].

The Hubble Tidal Term: We have shown [3,1] that there is an extra tidal term in Newton's law for the Kepler problem of planetary motion about mass M

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} + \frac{\ddot{a}}{a}\vec{r} \quad , \qquad (1)$$

created by the Riemann curvature tensor [2,3] in FL cosmology, using spherical coordinates (r,θ,φ). This extra term involves $\ddot{a}$ and $\dot{a}$ representing respectively the acceleration and velocity of $a(t)$ (Figure 1), where a is the FL scale factor of expansion, and the dots correspond to differentiation with respect to time. Re-expressing (1) in cosmological terms, the Hubble parameter $H=\dot{a}/a$ and the deceleration parameter q, gives

$$\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r} - qH^2\vec{r} \quad . \qquad (2)$$

Here q (Figure 2) is defined as $q = -\ddot{a}a/\dot{a}^2$, while $q \rightarrow q_o$ and $a \rightarrow a_o$ represent the value of q and a today. H_o is currently estimated to be H_o = 71 km s^{-1} Mpc^{-1} and the related age of the universe is 13.75 Gyr. The latter can be obtained from Figure 1 for a/a_o= 1 with the cosmological density parameter Ω_Λ = 0.71 where $\Omega_\Lambda=\Lambda/3H^2$ and Λ is the cosmological constant that is responsible for the acceleration [6]. Λ can also be thought of as representing a time-independent dark energy. Since q_o~ -1 then $\ddot{a}/\dot{a} = +\Lambda c^2/3$ in (1) holds for the current epoch, a fact supported be recent studies of supernovae [5].

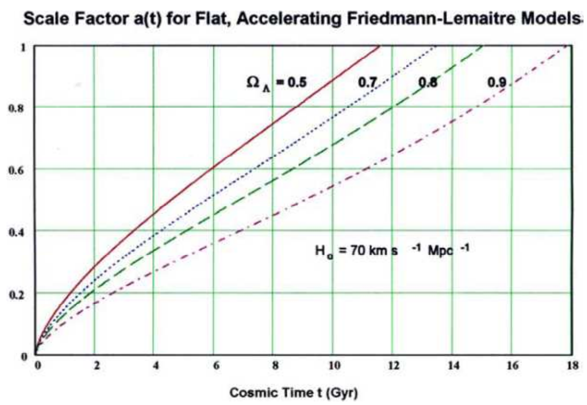

Figure 1. The FL scale factor $a(t)/a_o$.

The Binet and Clairaut Equations: Shortly after Newton introduced classical dynamics and gravitation theory, Binet and separately Clairaut devised techniques for solving the equations of motion. Binet's method consists of a re-definition or change of variables, in particular the substitution $u=1/r$ in (1) or (2). Clairaut's method can be thought of as the first integral of Binet's equation but introduces an angular variable φ which is important when considering the orbital precession of perihelia. Collectively then, $u=1/r(\varphi)$.

Hence, given the law of force (1) and the orbital angular momentum ℓ, Binet's equation yields the geometry of the orbit, $r(\varphi)$. Similarly, based upon the conservation of energy, and given the orbital shape $r(\varphi)$, energy E, and angular momentum ℓ, Clairaut's equation yields the potential $V(r)$ and the central force $F_c(r) = -\nabla V(r)$.

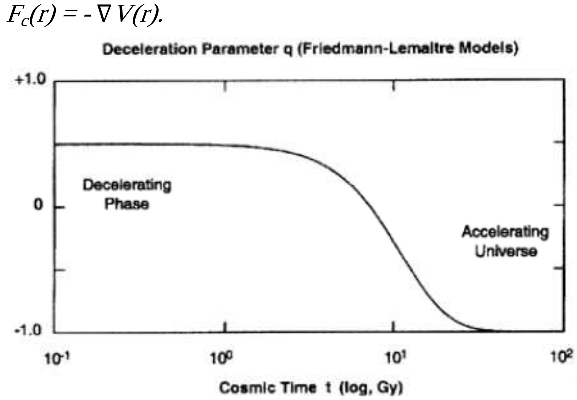

Figure 2. The deceleration parameter $q(t)$.

The method can be found in many places, but for General Relativity the problem of the precession of the perihelion of Mercury was originally calculated by Einstein and is elucidated by Synge [7,8]. Others include [9,10].

Thus Binet's equation can be derived from (1) using the conservation of angular momentum to substitute for the time derivative of φ. Let ℓ be the specific angular momentum $\ell = L/m$ where L is the total angular momentum and m is the mass involved. Then Binet's version of (1) reads

$$u'' + u = \frac{GM}{\ell^2} - \frac{\ddot{a}}{a} \cdot \frac{1}{\ell^2 \cdot u^3} \quad , \qquad (3)$$

and the primes represent differentiation with respect to r. In passing, other theories of gravity that introduce non-Newtonian effects and changes to General Relativity typically add terms to the right-hand-side of (3).

Multiplying (3) by u produces $(u')^2 + u^2$ on the left-hand-side, and Clairaut's equation for (3) is

$$(u')^2 + u^2 = \frac{2GM}{\ell^2} + 2\frac{\ddot{a}}{a} \cdot \frac{1}{\ell^2 \cdot u^2} + \frac{2 \cdot E}{\ell^2} \quad . \qquad (4)$$

This represents the Kepler problem in an expanding FL universe, as do (1) and (2).

Integrating (4) gives

$$\varphi + C = \int \frac{du}{\sqrt{\frac{2GM}{\ell^2} \cdot u + \frac{\ddot{a}}{a} \cdot \frac{1}{\ell^2 u^2} - u^2 + \frac{2E/m}{\ell^2}}} \qquad (5)$$

where C is a constant of integration. The general solution (5) can be expressed in terms of Jacobi elliptic functions and is described in [7].

Hubble Tidal Effect: We have described the additional term in (1) as a tidal effect and it is depicted in Figure 3. This represents the tidal perturbation (2) during cosmological evolution produced when the FL universe transitions to an accelerated phase in Figure 2 at $q = 0$. It occurs at the point of inflection (q=0) with a redshift $Z_* = 0.67$ when $\Omega_\Lambda = 0.7$ and $t(Z_*) = 7.2$ Gyr.

Furthermore, the FL universe becomes vacuum dominated at a redshift of $Z_{eq} = 0.33$ at the epoch $t(Z_{eq}) = t_{eq} = 9.2$ Gyr when the Solar System is currently believed to have formed 4.6 Gyr ago, assuming the age of the FL universe is $t_o = 13.8$ Gyr.

The correlation between the origin of the Solar System and the epoch at which the universe became dark-energy (cosmological constant) dominated at redshift Z_{eq}= 0.33, along with the tidal perturbation shown in Figure 3, both raise the interesting possibility that there is a cosmological influence of the universe on the physics and origin of local protoplanetary disks.

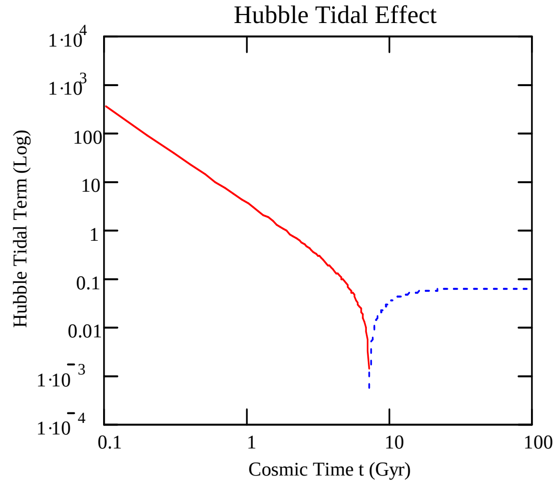

Figure 3. The Hubble tidal perturbation in evolutionary cosmology.

Protoplanetary Disks and the Tidal Term: Of course, the origin of the Solar System has been examined quite thoroughly dating back to Jeans. The more recent focus of attention has been the contribution of gravitational instabilities during the evolution of protoplanetary disks [11-12]. These include tidal triggering of star formation by the galaxy [13] and the formation of giant planets [14-18]. These do not, however, include the cosmological effect discussed here.

Conclusions: There is a need for the protoplanetary disk analyses to be extended to examine cosmological triggering mechanisms such as discussed here.

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