A MODEL FOR THE ELASTIC LIBRATION OF EUROPA'S ICE SHELL

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Introduction: Europa, one of Jupiter's larger moons, receives a great deal of attention due to its potential for hosting life. This ocean world is mysteriously shrouded by a thick ice shell. In order to reveal information about what is under Europa's ice, we examine its longitudinal libration. Measuring librational motion can reveal important information on a body's internal structure through constraints on its moment of inertia [1]. As most satellites are, Europa is tidally locked to its parent planet. Europa's ice shell, however, may be frictionally decoupled from the synchronously rotating interior by it's fluid ocean [2]. But the shell remains locked to the fluid interior by another mechanism. If the shell is reoriented relative to the tidal bulge, the shell experiences an elastic stress which increases its energy state and induces a restoring torque [3]. With a knowledge of the full time-dependent tide, an equation of motion for the shell can be developed to model its response to tidal forcing. Using a suite of calculation tools called SatStress [4] we can evaluate the elastic torque and derive the ice shell's equation of motion.

SatStress: Using tidal potential theory two varieties of stress on an icy satellite can be evaluated, one resulting from the time-dependent diurnal tide and another through non-synchronous rotation (NSR) of the shell [5]. SatStress was developed to calculate both forms of stress arising from tidal potential separately. We make one slight modification to the calculation by setting the stress equal to zero at our equilibrium position, which we have defined as perijove. This has the effect of assuming the ice shell has the same ellipsoidal shape as Europa's tidal bulge rather than assuming a spherical shell has been stretched over the satellite. We first examine relationship between NSR stress and the displacement angle of the shell from its equilibrium position.

Elastic Restoring Torque: Consider Europa with its tidal bulge aligned toward and away from Jupiter. Now allow the shell to slide over the bulge by an angle λ as in Fig. 1. Using SatStress's functions for NSR, we calculate the stress tensor associated with all colatitudes and longitudes in increments of one degree. We do this for λ for a full rotation of the ice shell.

From the stress tensors (σ_{ij}) we can calculate the strain tensors (σ_{ij}) at each point using Hooke's Law. The elastic energy density,

$$\varepsilon = \frac{\epsilon_{ij}\sigma_{ij}}{2}, \quad (1)$$

is integrated over the volume of the shell, and we arrive at the elastic energy associated with each displacement, λ, from equilibrium. We find that the energy difference

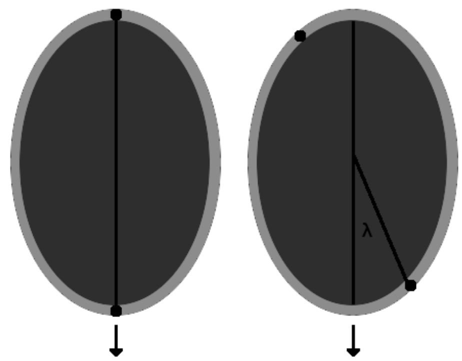

Figure 1: Europa with tidal bulge aligned toward Jupiter (arrow). Features on the ice shell surface are marked and displaced by an angle λ from equilibrium.

from the initial state is proportional to λ^2 for small angles (Fig. 2). We label the constant of proportionality P.

$$E(\lambda) = P\lambda^2. \quad (2)$$

The change of this energy with respect to angle is the elastic restoring torque, which is proportional to λ.

$$\frac{\delta E(\lambda)}{\delta\lambda} = \tau(\lambda) = C\ddot{\lambda} = 2P\lambda, \quad (3)$$

where C is the ice shell's rotational moment of inertia. These results for small displacements are consistent with Goldreich and Mitchell (2010) [3], but with SatStress we can accommodate large angular displacements. This torque, τ, is the elastic restoring torque.

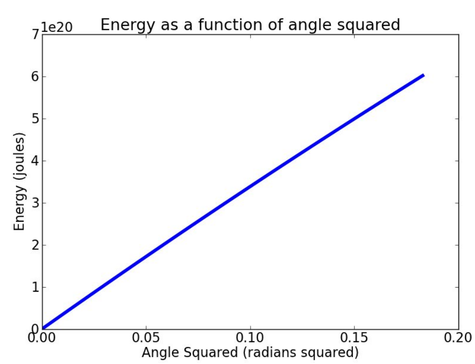

Figure 2: The displacement of Europa's ice shell by an angle λ provides an elastic energy proportional to λ^2.

Forced Libration: Using our knowledge about the ice shell's response to an angular displacement we next examine the shell's equation of motion over diurnal time scales. Allowing the tide to evolve with time, we use SatStress to track the elastic torque and integrate forward the equation of motion in order to derive the ice shells librational amplitude and frequency. To picture this, imagine Europa in the guiding center reference frame (Fig. 3). The tidal bulge nods back and forth as it follows Jupiter according to,

$$\theta(t) = 2e\sin(nt), \tag{4}$$

where e is the orbital eccentricity of Europa, and n is its mean motion. This nodding causes the diurnal stress which draws the shell forward by an angle, λ, causing a small NSR. The remaining stress on the shell is due to the angle, α, the shell is displaced from the tidal bulge,

$$\begin{aligned} \alpha(t) &= \theta(t) - \lambda \\ &= 2e\sin(nt) - \lambda. \end{aligned} \tag{5}$$

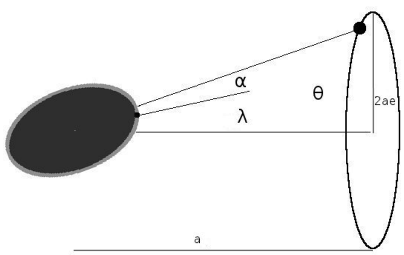

Figure 3: Reference frame centered on and rotating with Europa. In this frame Jupiter appears to bob back and forth on an ellipse.

Using α in place of λ in Eq. 3 we can write the equation of motion describing the oscillations of the ice shell.

$$\begin{aligned} C\ddot{\lambda} &= 2P\alpha(t) \\ &= 4Pe\sin(nt) - 2P\lambda. \end{aligned} \tag{6}$$

Eq. 6 is a differential equation for a forced oscillator having solutions of the form,

$$\lambda(t) = \gamma\sin(nt). \tag{7}$$

Taking the time derivative of this twice and plugging $\ddot{\lambda}$ and λ into Eq. 6 we can solve for γ, the forced libration amplitude,

$$\gamma = \frac{\frac{4Pe}{C}}{\frac{2P}{C} - n^2}. \tag{8}$$

If we set $\omega_0^2 = \frac{2P}{C}$ the equation becomes,

$$\gamma = \frac{2e\omega_0^2}{\omega_0^2 - n^2}, \tag{9}$$

where ω_0 is the free libration frequency *due to elastic restoring*.

Future Work: Our calculations thus far have focused on elastic ice shells of a single thickness. We would like to examine the effects on the libration due to varying the ice shell's thickness as well as it's rheology. A parameter in SatStress, Δ, is used to describe the elastic response of the ice.

$$\Delta = \frac{\mu}{\eta\omega}, \tag{10}$$

where μ, η, and ω are shear modulus, viscosity, and forcing frequency respectively [5]. When $\Delta \gg 1$ the ice shell will respond more fluidly. If the shell were to slip over the ocean, instead of producing a restoring torque, surface features on the shell will rise and fall as they pass over the tidal bulge. In the situation where $\Delta \ll 1$ the ice shell responds elastically. It is in this case where the shell will deform, store elastic energy, and induce a restoring torque [3]. Our study has begun using $\Delta = 0.1$, but we will continue to examine how this parameter effects the libration amplitude and frequency.

We also plan to test this technique with other satellites of the outer solar system. We expect similar models can be developed for Io, Titan, and Ganymede.

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