Influence of formation of temporary gravitational aggregates on ring viscosity

Yuki Yasui[1], Keiji Ohtsuki[1,2,3], Hiroshi Daisaka[4]. [1]Department of Earth and Planetary Sciences, Kobe University, Kobe 657-8501, Japan; [2]Center for Planetary Science, Kobe University, Kobe 650-0047, Japan; [3]Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO 80309-0392; [4]Graduate School of Commerce and Management, Hitotsubashi University, Tokyo 186-8061, Japan

Introduction: Viscosity in planetary rings arising from collision and gravitational interactions between particles governs the rate of their dynamical evolution and structure formation. Furthermore, recent studies show that Saturn's small moons orbiting just outside the main rings likely formed by accretion of particles radially spreading from the ring outer edge, thus the ring's spreading rate is also related to the formation of these moons.

The viscosity in Saturn's rings has been investigated with both theoretical and numerical approaches. In earlier theoretical studies the viscosity was estimated neglecting mutual gravitational forces between particles [1,2]. However, local N-body simulations including collision and mutual gravitational forces between particles [3-6] showed that, in optically thick rings, wake structures are formed due to their self-gravity. Observations by the Cassini spacecraft as well as various ground-based observations show results consistent with the existence of wake structures. Viscosity in such self-gravitating dense rings has been studied using local N-body simulations [7]. Numerical results show that the viscosity is significantly enhanced due to the effect of self-gravity in dense rings with gravitational wake structures. From their numerical results, Daisaka et al. [7] derived a formula of the viscosity in self-gravitating dense rings given by $\nu \sim CG^2\Sigma^2/\Omega^3$, where G is the gravitational constant, Σ and Ω are the surface density and angular velocity of the ring, and C is a correction factor that depends on the ratio of particles' Hill radius to their physical size. However, the range of parameters studied by Daisaka et al. [7] was somewhat limited; for example, they assumed that particles are smooth spheres, and did not examine effects of surface friction of particles on the viscosity. Also, they did not examine the viscosity in the outer part of the rings, where temporary gravitational aggregates can be formed.

In the present work, using local N-body simulations, we examine viscosity of planetary rings consisting of spinning, self-gravitating particles for a wide range of parameters, including the cases of dense rings with temporary aggregate formation.

Method: We adopt the method of local N-body simulation, and use a code based on Daisaka & Ida [5] and Daisaka et al. [7]. We erect a rotating Cartesian coordinate system with origin at the center of the square simulation cell that moves on a circular orbit with semi-major axis a_0 at the Keplerian angular velocity Ω. We calculate directly gravitational forces between particles using GRAPE-7, which is a special-purpose hardware for calculating gravitational forces, and orbits of particles are integrated with the second-order leapfrog method. In the present work, we assume that all particles have identical sizes and particle radius is $1\mathrm{m}$. Following Salo [3], in most cases we set the size of the simulation cell so that $L \geq 4\lambda_{\mathrm{cr}}$, where $\lambda_{\mathrm{cr}} = 4\pi^2 G\Sigma/\Omega^2$ is the critical wavelength for axisymmetric gravitational instability.

When collisions between particles are detected, velocity changes are calculated based on the hard-sphere model including surface friction. Velocity changes are described in terms of the normal and tangential restitution coefficients, ε_{n} and ε_{t}, where $0 \leq \varepsilon_{\mathrm{n}} \leq 1$ and $-1 \leq \varepsilon_{\mathrm{t}} \leq 1$. (Perfectly smooth spheres have $\varepsilon_{\mathrm{t}} = 1$.) We calculate ring viscosities based on the formulation derived by Tanaka et al. [8]. Deriving an expression for angular momentum flux in terms of changes in particle orbital elements and assuming a quasi-steady state in a local ring region, Tanaka et al. [8] showed that the averaged viscosity in the ring region can be obtained by calculating energy dissipation due to inelastic collisions using N-body simulation.

Results: In the case of tenuous self-gravitating rings, we found that viscosity is determined by particles' random velocity, as in the case of tenuous non-gravitating rings. Effects of surface friction on viscosity was found to depend on the importance of particles' mutual gravity relative to collisions, which can be expressed in terms of the ratio of the mutual Hill radius of colliding particles to their physical size (r_{h}). When effects of ring's self-gravity is weak $(r_{\mathrm{h}} \sim 0.5)$ and particles' random velocity is determined by inelastic collisions, inclusion of surface friction slightly reduces both random velocity and viscosity. When $r_{\mathrm{h}} \sim 1$ and gravitational encounters play a major role in particle velocity evolution, inclusion of surface friction results in the increase of viscosity, since the viscous heating increases to balance with increased energy dissipation at collisions due to surface friction. However, the degree of enhancement in viscosity due to surface friction is rather small, and we found that effects of varying restitution coefficients on ring viscosity are not significant. On the other hand, viscosity is significantly enhanced in dense self-gravitating rings where gravitational wakes are formed [7]. We found that viscosity is larger when collisions are more dissipative, for example, by inclusion of surface friction. However, the enhancement of viscosity due to this effect is only a factor of a few or so, and this is a minor effect as compared to the significant increase due to the effect of gravitational wakes. Following Daisaka et al. [7], we assumed that the dependence of viscosity on optical depth can be approximated by that for a self-gravitating disk and evaluated the correction factor C using our numerical results of N-body simulation. We found that the expression for the correction factor for dense rings with gravitational wakes is similar to the one obtained by the previous work [7], the difference of a factor two in the numerical coefficient being explained by the difference in restitution coefficients adopted.

On the other hand, in the case of outer rings where temporary aggregates are formed due to self-gravity, we found that the above expression overestimates the viscosity values (Fig.1a), thus we derived a revised expression for such a case. We confirmed that the set of semi-analytic expressions we obtained well reproduce our numerical results for the entire range of parameters examined here (Fig.1b).

Acknowledgments: This work was supported by JSPS, and NASA's Planetary Geology and Geophysics Program and Outer Planets Research Program.

References: [1] Goldreich, P., and S. Tremaine 1978, Icarus 34, 227-239. [2] Wisdom, J., and S. Tremaine 1988, Astron. J. 95, 935-940. [3] Salo, H. 1995, Icarus 117, 287-312. [4] Richardson, D. C. 1994, Mon. Not. R. Astron. Soc. 269, 493-511. [5] Daisaka, H. and S. Ida 1999, Earth Planets Space, 51, 1195-1213. [6] Ohtsuki, K. and H. Emori 2000, Astron. J. 119, 403-416. [7] Daisaka, H., H. Tanaka, and S. Ida 2001, Icarus 154, 296-312. [8] Tanaka, H., K. Ohtsuki, and H. Daisaka 2003, Icarus 161, 144-156. [9] Bridges, F., A. P. Hatzes, and D. N. C. Lin 1984, Nature 309, 333-335.

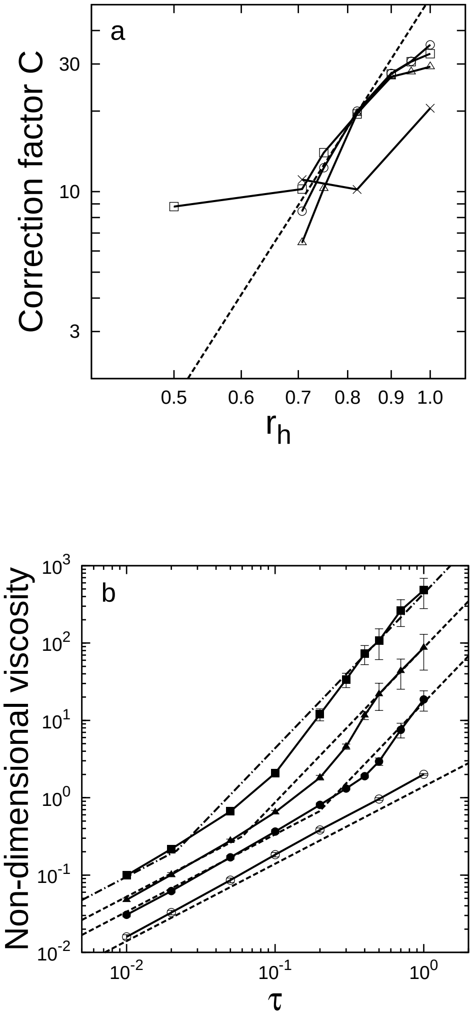

Figure 1: (a) Correction factor $C \equiv \nu / \left(G^2\Sigma^2/\Omega^3\right)$ as a function of r_h. Solid lines with symbols represent results obtained from our N-body simulation ($\varepsilon_\mathrm{n} = \varepsilon_\mathrm{n}\left(v\right)$, $\varepsilon_\mathrm{t} = 0.5$, where $\varepsilon_\mathrm{n}\left(v\right)$ is the velocity-dependent coefficient based on [9]) with optical depth τ =0.2 (crosses), 0.5 (triangles), 0.7 (circles), and 1 (squares). Dashed line represents the relation $C_\mathrm{wake} = 53 r_\mathrm{h}^5$, which well reproduces numerical results for rings with gravitational wakes. (b) Comparison between semi-analytic results (dashed lines and dot-and-dashed line) and N-body simulation (solid lines). The four lines correspond to r_h =0.5, 0.71, 0.82, and 1, from bottom to top. For the semi-analytic results shown with the dashed lines, we used $C_\mathrm{wake} = 53 r_\mathrm{h}^5$. On the other hand, $C_\mathrm{agg} = 30$ is used for the case with $r_\mathrm{h} = 1$, where temporary gravitational aggregates are formed (dot-and-dashed line).