HOW DOES NONHYDROSTATICITY AFFECT THE DETERMINATION OF ICY SATELLITES' MOMENT OF INERTIA? P. Gao[1] and D. J. Stevenson[1], [1]California Institute of Technology, Pasadena, CA, United States (pgao@caltech.edu).

Introduction: Current models of icy satellite formation and evolution depend on the accuracy with which we determine their interior structures. These can be inferred from their moments of inertia (MOI), which can be estimated from *in situ* gravitational field measurements by spacecraft. The primary method of estimation is the Radau-Darwin approximation (RDA) [1], which relates the MOI to the degree 2 response of the body to rotation and tides (gravitational coefficient) J_2 and C_{22}. This method makes the assumption that the body is in hydrostatic equilibrium and also ignores the effects of large density variations. It has been applied to several large icy satellites in the outer Solar System, including Titan [2], Callisto [3], and Ganymede [4]. Interpretations of the correlation between their MOIs and orbital distances lend credence to the "gas starved disk" model [5]. However, hydrostatic equilibrium is not guaranteed ([2], [3], [4], [5]), so it is prudent to assess the impact of nonhydrostatic structures on the accuracy of RDA. We use a simple model to show that relatively small departures from hydrostatic equilibrium can result in errors of up to 10% in the RDA-calculated MOI, and faster-rotators, with all else being equal, require a greater departure in order to exhibit the same error, thus casting doubt on at least part of the claimed differences in Titan, Callisto, and Ganymede's MOIs.

Icy Satellite Model: Our model consists of a layer of water ice with density 1g/cc overlaying a rock core with density 3g/cc. This simple arrangement allows us to isolate the essential physics of the problem. We further assume that the only departure from sphericity is a rotational bulge, since the (larger) tidal distortion involves the same physics and can be readily superimposed. The degree 2 zonal distortions in the shape and gravitational field of the satellite is described by two oblateness factors ε_o and ε_c, for the surface of the satellite, and the surface of the core, respectively. These oblateness factors are small for all bodies of interest. This simple model allows us to completely determine the oblateness factors and J_2 in terms of the fractional core radius, the ratio between the ice and core densities, the rotation rate, the mean radius, and the mass by assuming that both the satellite and core surfaces are equipotentials under the condition of hydrostatic equilibrium.

The RDA predicts a one-to-one correspondence between MOI and J_2 for a specific rotation. Nonhydrostatic structure destroys this one-to-one correspondence. RDA can also fail if there are large density variations, but we found this effect to be negligible even for the factor of three difference between core and mantle densities in our model. The nonhydrostatic effect can be quantified by introducing a small change $\Delta\varepsilon$ in the hydrostatic oblateness factors for core and satellite surfaces; we then relate these to the amount of stress these imply for the satellite and evaluate whether or not they are feasible. The stresses are estimated via:

$$\sigma_o = \rho_o g_o |\Delta \epsilon_o| R_o \qquad \sigma_c = (\rho_c - \rho_o) g_c |\Delta \epsilon_c| R_c \qquad (2)$$

Where σ_x is stress; ρ_x is density; $\Delta\varepsilon_x$ is the difference in oblateness factor between the hydrostatic and nonhydrostatic cases; g_x is the gravitational acceleration, R_x is the radius; and $x = o, c$ indicates satellite surface and core surface, respectively.

Applications to Real Satellites: We applied our model to Titan, Callisto, and Ganymede by using their published values of mass, radius, and rotational period in our equations. It should be noted that the real satellites are far more complicated internally than our model, but this is irrelevant here, as we are interested in an order-of-magnitude estimate of the nonhydrostatic effects and not a precise value.

Titan. Iess et al. [2] reported Titan's MOI to be 0.3414 ± 0.0005, suggesting partial differentiation or complete differentiation with a low-density, hydrated core [6]. However, recent work by O'Rourke and Stevenson [7] has shown that a compositional gradient likely existed in early Titan, which would've prohibited convection to the point where the ice near the center would've melted due to long-lived radioactive isotopes, leading to complete differentiation; the trapped radiogenic heat would've likely dehydrated the core as well in the age of the Solar System [6]. Thus, Titan is likely completely differentiated with a rocky core, even if it started out partially differentiated during accretion. The MOI for a completely differentiated Titan is ~0.31, similar to that of Ganymede. We find in our calculations that, in order to produce a 10% error in the measured MOI (enough to shift it to ~0.31), the degree 2 nonhydrostatic structure, if purely in the form of surface topography, needs to have a maximum amplitude of ~20 m on the satellite surface, or ~30 m on the core surface, corresponding to load stresses of ~0.3 and ~1 bar(s), respectively.

Callisto. Anderson et al. [3] reported Callisto's MOI to be 0.3549 ± 0.0042, suggesting a similar structure to Titan, and possibly the same kind of disagreement between its interior models and its measured MOI. Our work shows that the maximum amplitude of nonhydrostatic structure needed to produce a 10% error in MOI and the corresponding load stresses are also equal to that of Titan.

Ganymede. Anderson et al. [4] reported Ganymede's MOI to be 0.3105 ± 0.0028, suggesting a completely differentiated body. This interpretation is further supported by the existence of an intrinsic magnetic field, indicating an iron or iron-sulfide core [4]. To produce a MOI error of 10% here, the nonhydrostatic structure must have an amplitude of ~100 m on the satellite surface, or ~200 m on the core surface, corresponding to load stresses of ~2 or ~6 bars, respectively.

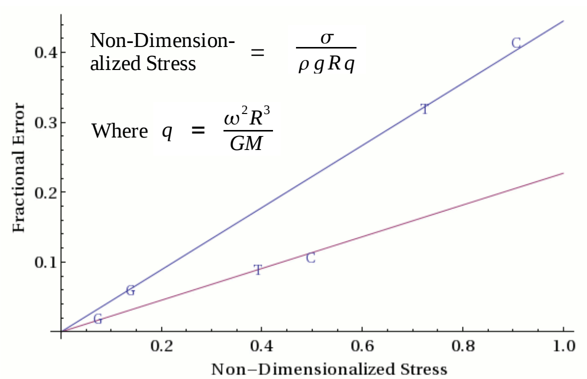

Figure 1. The fractional error in MOI vs. the non-dimensionalized surface (blue line) and core (red line) stresses using Titan's model core fraction (similar to those of the other bodies in consideration). The position of the letters indicate the fractional errors caused by 1 bar of nonhydrostatic stress on the surface and core of Ganymede (G), Titan (T), and Callisto (C). ω is the angular rotation rate, M is the satellite mass, and G (in the q equation) is the gravitational constant.

Implications: It is apparent that the same amount of departure from a hydrostatic figure would have a greater effect on Titan and Callisto's MOIs than on Ganymede's. This is further quantified in Figure 1, where 1 bar of nonhydrostatic stress on the surface or the core results in a greater fractional error in the MOI of Titan and Callisto than that of Ganymede. Thus, it is valid to suggest that all three satellites are almost fully-differentiated, but the incorrectly-inferred MOIs for Callisto and Titan are higher at least partially due to invalid assumptions of hydrostatic equilibrium in RDA, the method used to determine them.

It should be noted that the error caused by degree-2 nonhydrostatic stress would tend to result in a higher MOI than the real value, rather than lower, due to true polar wander maximizing the polar MOI through reorientation, i.e. the error would shift a true MOI of ~0.31 to ~0.34 rather than ~0.28.

Sources of Nonhydrostatic Structures: The sources of nonhydrostaticity can be divided into two categories: primordial and current. Primordial sources include fossil equatorial bulges that froze into shape before the satellite synchronized its rotation; an extreme example of this is Iapetus, where a fossil bulge corresponding to a 16 hour rotation period exists even though its current rotation period is ~80 days [8]. The feasibility of primordial sources lies in whether the nonhydrostaticity can survive to the present day; Iapetus' shape is preserved likely due to a thick lithosphere holding up the fossil bulge [8]; can a much smaller fossil bulge be present either on the surface or at the core-mantle boundary of larger icy satellites?

Current sources include stresses caused by ongoing convection [9], which can be on the order of 1 bar for large icy satellites [10]. However, convection will not preferentially affect degree 2, so their presence is potentially revealed in other harmonics.

Observed Nonhydrostatic Structures: Iess et al. [1] reported nonhydrostatic geoid variations of up to 19 m on Titan, similar to our results; the sources of these variations are unknown. Mass anomalies have also been detected on Ganymede that appear to correlate with the surface variations in brightness [11]. These observations, combined with our work, suggest that even these "minor" nonhydrostatic structures can create relatively large errors in the measured MOIs, and subsequent interior models. Thus, more data is needed to better characterize these nonhydrostatic structures in order to help determine the true MOIs of these bodies.

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