COMET ROTATIONAL RELAXATION AND INTERIOR STRESSES AND LOADS. D.J. Scheeres, *U. Colorado, Boulder (scheeres@colorado.edu)*, S.A. Jacobson, *U. Colorado, Boulder*.

When comet nuclei are subject to outgassing they can be excited into a non-uniform rotation state. This causes time-varying internal stresses and strains in the nucleus which are then damped over time towards a uniform rotation state. This abstract explores the implications of these excitations and dissipations for comet nuclei, and show that they may be related to the still unexplained "comet bursting" phenomenon.

Specifically, we analyze the interior stresses and loads that a nucleus will experience when in an excited state and show how they evolve as its excess rotational energy dissipates. The interior pressure and tensile stresses required to keep the body from failing all decrease as the excess energy is dissipated. In contrast the nucleus-fixed loads that the body experiences increase with relaxation for a prolate body. This could result in a propensity for prolate nuclei to undergo bursting phenomenon. Oblate nuclei do not undergo a similar increase in body-loads when they relax, and instead experience their maximum loads at their maximum excitement level.

Comet Rotation State Excitation/Relaxation Balance It has now been clearly demonstrated, both analytically and observationally, that comet outgassing can change the spin state of a nucleus, and hence can also excite the nucleus into a non-principal axis spin state [1, 2, 3, 4, 5]. Whether a comet remains in an excited state or relaxes into principal axis rotation about its largest moment of inertia depends entirely on the rate of internal dissipation within the nucleus. Constraints and measurements of this dissipation rate would provide details on the interior strength and structure of a comet nucleus. Independent of what the energy dissipation rate of a comet nucleus is, it is clear that they experience such dissipation. Analytical studies by Neishtadt et al. [2] clearly show that without dissipation the general trend for most cometary nuclei subject to outgassing is to progress towards a highly excited state, which is not commonly seen in cometary nuclei.

The combination of random rotational excitation and dissipation has been shown to lead to a steady increase in a rotating body's spin rate [6]. The mechanism is simple. While random torques along the rotation vector average to zero to first order, random torques perpendicular to the angular momentum increase the total angular momentum and, when they eventually damp to uniform rotation, necessarily increase the rotation rate. Thus, excited and dissipative bodies will naturally increase their spin rates over time, if subject to random outgassing torques. If subject to systematic torques, then the net change may either be an increase or decrease and would be much more visible over short time spans.

This abstract restricts itself to nuclei with an axis of symmetry, for simplicity, although more general mass distributions can also be treated similarly. Assume an ellipsoid defined by its axis of symmetry semi-major axis, α, and its transverse, equal semi-major axes, β. Its mass-normalized moments of inertia about its axis of symmetry and about its transverse axis then equal $I_a = 2\beta^2/5$ and $I_t = (\alpha^2+\beta^2)/5$, respectively. If $\alpha > \beta$ the body is prolate and its minimum energy state is to rotate about an axis perpendicular to its axis of symmetry. If $\alpha < \beta$ the body is oblate and its minimum energy state is to rotate about its symmetry axis. In the following we assume the nucleus has a constant angular momentum of magnitude H, and that the angle between its symmetry axis and the plane perpendicular to its angular momentum vector is δ. The angular velocity vector is then

$$\boldsymbol{\omega} = \omega_a \hat{\boldsymbol{a}} + \omega_t \left(\cos\theta \hat{\boldsymbol{t}} + \sin\theta \hat{\boldsymbol{t}}_\perp\right) \quad (1)$$

$$\omega_a = \frac{H\sin\delta}{\sqrt{I_a^2\sin^2\delta + I_t^2\cos^2\delta}} \quad (2)$$

$$\omega_t = \frac{H\cos\delta}{\sqrt{I_a^2\sin^2\delta + I_t^2\cos^2\delta}} \quad (3)$$

where $\hat{\boldsymbol{a}}$, $\hat{\boldsymbol{t}}$, $\hat{\boldsymbol{t}}_\perp$, are the nucleus fixed axes along the symmetry and transverse axes, respectively, and θ is the angle the angular velocity makes in the plane defined by the transverse axes. While the angular velocity about the symmetry axis is constant (neglecting changes due to dissipation), the angular velocity about the transverse axis continually rotates at a constant rate $\dot\theta$ about the symmetry axis, and is the motive cause for the energy dissipation. As the nucleus dissipates energy, if it is prolate $\delta \rightarrow 0^\circ$ while if it is oblate $\delta \rightarrow 90^\circ$.

Internal Nucleus Stresses and Loads The internal loads due to self-gravity and rotational dynamics for an ellipsoidal body can be reduced to

$$\boldsymbol{g} = [\boldsymbol{G} - \boldsymbol{A}]\cdot \boldsymbol{r} \quad (4)$$

where the acceleration term is $\boldsymbol{A} = \dot{\tilde{\boldsymbol{\omega}}} + \tilde{\boldsymbol{\omega}}\cdot\tilde{\boldsymbol{\omega}}$, and the gravitational term is $\boldsymbol{G} = G_\alpha \hat{\boldsymbol{a}}\hat{\boldsymbol{a}} + G_\beta(\hat{\boldsymbol{t}}\hat{\boldsymbol{t}} + \hat{\boldsymbol{t}}_\perp\hat{\boldsymbol{t}}_\perp)$, $\tilde{\boldsymbol{\omega}}$ is the skew-symmetric cross-product form of the angular velocity and $\boldsymbol{r}$ is a location inside the body. We note that the angular velocity terms are time-varying in this formulation. The volume-averaged stress tensor can be computed from this formulation: $\boldsymbol{\sigma} = \frac{\rho}{V}[\boldsymbol{G}-\boldsymbol{A}]\cdot\int\int\int_{\mathcal{E}} \boldsymbol{r}\boldsymbol{r}dV = \frac{\rho}{5}[\boldsymbol{G}-\boldsymbol{A}]\cdot\boldsymbol{\mathcal{I}}$ where $\boldsymbol{\mathcal{I}} = \alpha^2\hat{\boldsymbol{a}}\hat{\boldsymbol{a}} + \beta^2(\hat{\boldsymbol{t}}\hat{\boldsymbol{t}} + \hat{\boldsymbol{t}}_\perp\hat{\boldsymbol{t}}_\perp)$. It is interesting to note that the resulting stress tensor $\boldsymbol{\sigma}$ is symmetric, although to show this the time derivative of the angular velocity solution must be inserted.

The volume-averaged principal stresses can be found analytically and used to evaluate the internal pressure and the Drucker-Prager failure criterion. Following Holsapple [7] we define the Drucker-Prager failure criterion as

$$\frac{1}{\sqrt{6}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} + 3sp \le k$$

where $\sigma_1 \ge \sigma_2 \ge \sigma_3$ are the principal stresses, k is the cohesive shear stress for failure at zero pressure, $3p = \sigma_1 +$

$\sigma_2 + \sigma_3$ and $s = 2\sin\phi / \left[\sqrt{3}(3 - \sin\phi)\right]$, where ϕ is the friction angle and is taken to be $45°$ by Holsapple. In Fig. 1 we plot k and p for a prolate and oblate nucleus as a function of the dissipation angle δ, showing that the interior pressure and stresses all reduce with relaxation.

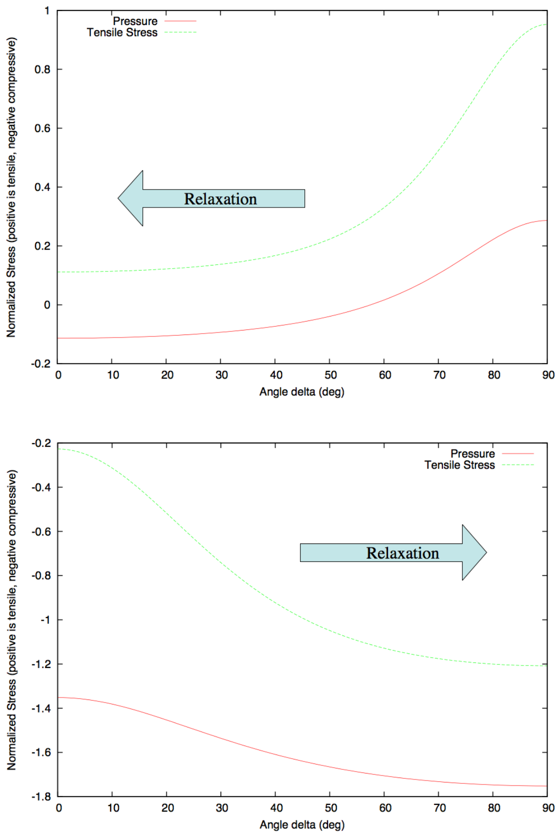

Figure 1: Pressure and Drucker-Prager Tensile stress to prevent failure as a function of relaxation angle δ. Top: a prolate 2:1 nucleus. Bottom: an oblate 1:2 nucleus.

While the total and maximum principal stresses all decay with relaxation, we note that the loads due to these principal stresses are always acting in different directions and thus do not constitute a systematic load across the body. If, instead, we consider the load that the nucleus feels in a fixed direction within the body we see that these may actually increase with relaxation. This only occurs for prolate bodies relaxing to their minimum energy state, oblate bodies feel a decreasing load across their equator as they relax. To capture this we consider the time-averaged load along the nucleus-fixed symmetry and transverse axes, plotted in Fig. 2.

Model for Comet Bursting This leads to a notional model for comet nucleus bursting that can explain some aspects of the observed effect assuming that a comet's nucleus is excited into a complex rotational state. Due to the interior physics of these bodies they are assumed to be able to sustain tensile

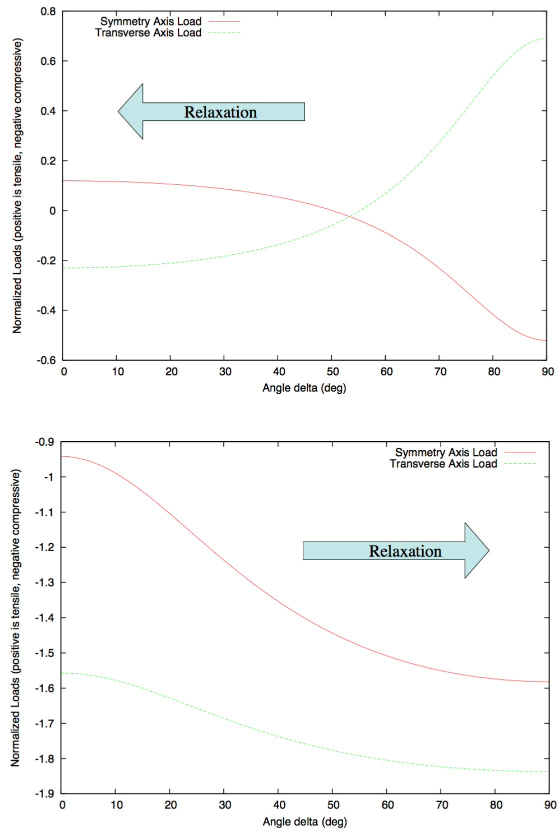

Figure 2: Symmetry and averaged transverse axis loads as a function of relaxation angle δ. Top: a prolate 2:1 nucleus. Bottom: an oblate 1:2 nucleus.

loads. If not, then they should fail as soon as they are excited beyond their tensile limit, which would generally occur around perihelion and is not seen. The rotating body commences its dissipation and relaxes to uniform rotation about its smallest axis for a prolate body. As this process occurs the interior stress orientations vary with time in a cyclic fashion perpendicular to the symmetry axis. Simultaneously, as the body continues to relax the axial load along the symmetry axis grows for a prolate body. The existence of a body-relative load that increases with relaxation opens the door for a delayed failure mechanism driven by dissipation. The actual interior stresses are not aligned and yield a variety of phased directions perpendicular to the symmetry axis as a function of time. They do represent a unified stress along the symmetry axis, which is the natural axis for initial failure and fission motion.

References: [1]: Samarasinha et al., Icarus 116: 340-358 (1995). [2]: Neishstadt et al., Icarus 157: 205-218 (2002). [3]: Belton et al., Icarus 175: 181-193 (2005). [4]: Belton et al., Icarus 213: 345-368 (2011). [5]: Knight et al., Astronomical J. 141:2 (2011). [6]: Scheeres et al., Icarus 147: 106-118 (2000). [7]: Holsapple, Icarus 187: 500-509 (2007).