FISSION LIMITS FOR BIFURCATED ASTEROIDS: THE CASE OF KLEOPATRA Masatoshi Hirabayashi[1], Daniel J. Scheeres[1], [1]University of Colorado at Boulder, CO 80309-0429, USA; Masatoshi.Hirabayashi@Colorado.edu

Abstract: Based on radar and optical measurement of Kleopatra we show that it is spinning near, but less than, its fission limit. Our method is also applied to contact binaries and asteroids to map out their spin limits before they experience internal tension and are susceptible to spin fission.

Introduction: Because of its shape, mass, and spin rate, some asteroids may be close to their fission limit. This limit may be strongly related to their shape; to compute the self-gravity it is necessary to take into account precise computation of the mutual attraction of the body. The current study obtains the rotation period at the fission limit and the normal stress on cross-section perpendicular to the minimum principle axis by calculating the 4th order polyhedral mutual gravity ([1], [2]) and shows that Kleopatra may be close to its fission limit. In the conference session, we will discuss 5 additional cases: Itokawa [3], Eros[4], Ida [5], 1996HW1 [6], and 1999KW4[7].

Fission Limit: Self-gravity and centrifugal forces due to rotation cause an arbitrary cross-section of an asteroid to have stress on it because of the law of action and reaction. As the rotational angular momentum of an asteroid increases, the centrifugal force may also increase, while the self-gravity keeps constant. Once the stress reaches beyond zero, the body is susceptible to undergoing fission at a particular cross-section. We call this transition point the "fission limit". It is not necessary for a body to fission; however, at spin rates higher than the fission limit, tensile strength is needed to keep the body together (note that we do not consider failure criterion here).

Mass, Density, Shape, and Rotation Period of Kleopatra: Ostro et al.[8] constructed a three dimensional shape model of Kleopatra based on radar observations with an equivalent radius of $54.3km$. Descamps et al. [9] computed the equivalent radius of the body as $67.5 \pm 1.1km$, based on IR observations, and the mass as $4.64 \pm 0.02 \times 10^{18}kg$, based on the two satellites orbiting around it. To compute the stress we refer to Ostro et al.[8] for the shape model and Descamps et al.[9] for the mass and the equivalent radius, scaling the Ostro model to the new size. Using the mass and the equivalent radius, we define the range of density to be $3.6 \pm 0.4g/cm^3$, where the mass is assumed to be constant, $4.64 \times 10^{18}kg$. As shown in Figure.1, Kleopatra looks like a dog-bone and is stretched along its minimum principle axis with bulbous ends. The rotation period is fixed to be $5.38528hour$. The following discussion deals with 5 density cases: 3.2, 3.4, 3.6, 3.8, and $4.0g/cm^3$. Note that varying the density causes the scale of the shape to change.

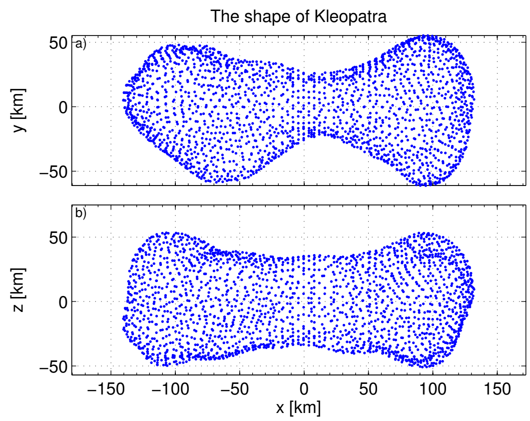

Figure 1: Kleopatra projected onto XY plane (Upper figure) and XZ plane(Lower figure)

Force Computation: The normal stress is found by dividing the internal force by the area of the cross-section perpendicular to the minimum principle axis of an asteroid, therefore it can be considered to be the averaged normal stress on the cross-section. We consider the total action and reaction force across the plane divided by its area from the self-gravity and the centrifugal force to compute the stress. Since slicing a body causes us to have the center of mass which is close to, but does not lie along, the principle axis, we show the component perpendicular to the cross-section only.

The gravity force is calculated from the shape model of Kleopatra. The dataset includes the three dimensional position of vertices and their order. Slicing an asteroid through an arbitrary plane perpendicular to the minimum principle axis, we generate two bodies by making new vertices and orderings to express a new plane. Then, we compute the polyhedral mutual gravity force between the two bodies using Legendre polynomial function expansion ([1],[2]). It is necessary to consider high order potential terms for this problem because the two bodies are close to each other, as shown in Figure.2, which indicates the normal stress along the long axis for the nominal density of $3.6g/cm^3$. Higher order computations take an increasingly long simulation time. For this reason, we compromise and set the highest order to be 4th order. Based on other computations, we believe that the 4th order computation is close to the convergence limit.

Results: The combined rotation period and density at the fission limit is indicated in Figure.3 by the non-

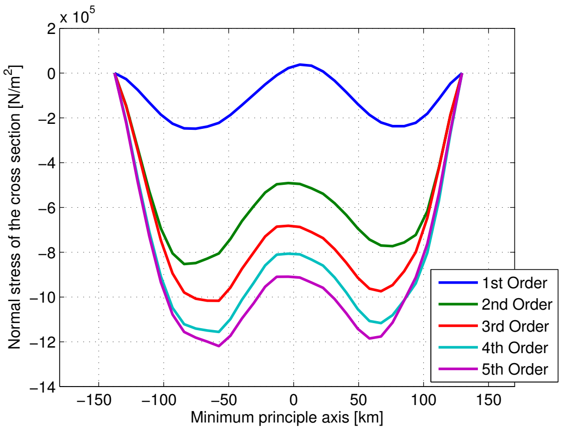

Figure 2: The result of normal stress due to different orders of the mutual gravity computation.

dimensional value α, defined to be $\omega^2/G\rho$, where G is the gravity constant, ω is the spin rate, and ρ is the density. The blue line is α^*, defined as the limiting spin rate/density combination that yields a zero stress across a body cut at that location. For the Kleopatra shape we see that the body is most likely to experience tension across its center, and that it is currently close to, but not at, this limiting spin rate. In addition, since the density is hidden by the normalization, α^* is given only by the shape and size. In Figure.3, we show only the case of $3.6g/cm^3$ because the value itself is independent of the density. Note that only the scale of the shape is different for each density. The minimum value is 0.542 at $x = 4.811km$. On the other hand, the red line is $\alpha_{re} = 0.437$, the value given by the physical conditions of mass equal to $4.64 \times 10^{18}kg$, density equal to $3.6g/cm^3$, and spin period equal to $5.38528hour$.

The normal stress is given by dividing the internal force by the area of the cross-section; therefore, it can be considered to be the averaged normal stress on the cross-section. Figure.4 shows the normal stress computed over the range of densities from $3.2g/cm^3$ to $4.0g/cm^3$. All the cases in the figure have extrema around the middle of the body, which means that the centrifugal force plays a more important role. Again, since we assume the mass to be constant and change its density, the scale of Kleopatra is also changed. If the density is set to be $3.2g/cm^3$, the size is 1.269 times larger than the model given by Ostro et al. [8], while for the density of $4.0g/cm^3$ we magnify the model by the factor 1.178. It is reasonable that the lower density causes the normal stress to be more critical than the higher density because the gravity force gets lower due to the lower density while the centripetal forces increase due to the larger volume.

Conclusion: Giving the rotation period at the fission limit and the normal stress along the minimal principle axis from the polyhedral mutual gravity, we showed that at the current condition, the middle part of Kleopatra may be close to the fission limit. In the session, we will discuss the case of Kleopatra in addition to other 5 asteroids: Itokawa [3], Eros[4], Ida [5], 1996HW1 [6], and 1999KW4[7].

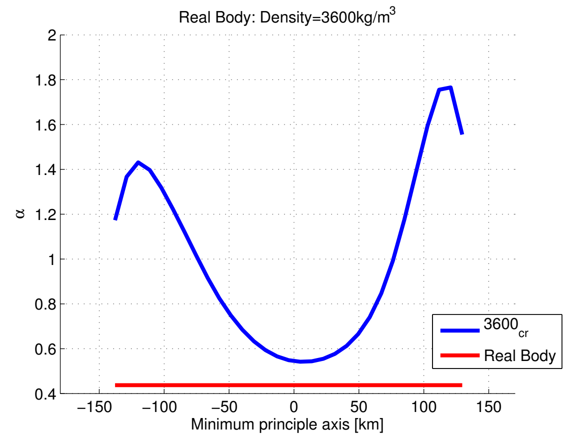

Figure 3: $\alpha = \omega^2/G/\rho$. The blue line is α^*, the case of the critical rotation period. The minimum value is 0.542 at $x = 4.811km$. The red line, on the other hand, is $\alpha_{re} = 0.437$, the case where the density is $3.6g/cm^3$ and the rotation period is $5.38528\ hour$. $\alpha^* - \alpha_{re}$ is dependent on the density and spin rate.

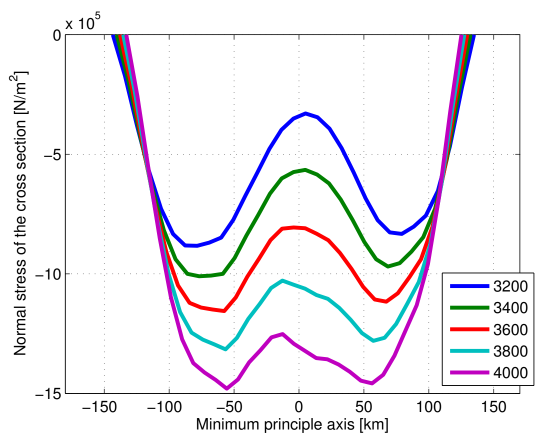

Figure 4: Normal stress acting on the cross-section. The gravity force and the centrifugal force are considered.

References: [1] R. A. Werner, et al. (2005) *Springer* 91:337. [2] E. G. Fahnestock, et al. (2006) *Celestial Mech Dyn Astr* 96:317. [3] A. Fujiwara, et al. (2006) *Science* 312:1330. [4] J. K. Miller, et al. (2002) *Icarus* 155:3. [5] M. J. S. Belton, et al. (1995) *Nature* 374:785. [6] C. Magri, et al. (2011) *Icarus* 214:2011. [7] (2006) *Science* 314:1276. [8] S. J. Ostro, et al. (2000) *Science* 288:836. [9] P. Descamps, et al. (2011) *Icarus* 211:1022.