WHY DO WE SEE THE MAN ON THE MOON? Oded Aharonson[1,2], Peter Goldreich[1], Re'em Sari[3], [1]Division of Geological and Planetary Sciences, California Institute of Technology, MC 150-21, Pasadena, CA 91125, USA (oa@caltech.edu); [2]Center for Planetary Science, Weizmann Institute of Science, Rohovot, Israel; [3]Racah Institute of Physics, Hebrew University of Jerusalem, Israel.

Introduction

The face of the Moon that faces Earth is largely covered by dense, topographically low, dark mare basalts. In some cultures, the pattern of these compositionally distinct plains is reported to resemble the image of a man's face, which continuously points towards Earth in the current spin-orbit resonance. Internal dissipation causes the Moon to spin about its axis of maximum moment of inertia and point its axis of minimum inertia towards the Earth. The axis of intermediate moment points along the orbit. This leaves the two options of the Moon facing either its current near-side or far-side towards the Earth. Both configurations are local potential energy minima, because the energy is dominated by the second moment of the Moon's mass distribution, while higher moments break this symmetry.

The potential energy minima differ by $\delta U \sim 2.4 \times 10^{17}$ J, and the two potential energy maxima between them differ by $\Delta U \sim 7.2 \times 10^{16}$ J. If the Moon were to reverse its orientation such that the current far-side would face the Earth, the total potential energy of the system would be slightly lower than at present. The Moon is currently "stranded" in an energy state higher than the global minimum by an amount that is roughly $R_M/a \sim 1/240$ times the energy barrier separating these minima. However, as shown below, the choice of minima depends not on the difference in minima, δU, but rather on the difference in maxima ΔU.

Dynamical Evolution

In order to understand how the Moon arrived at its current state, we analyze a simplified system that exhibits the relevant dynamics. The system consists of damped motion of a particle in an approximate potential, neglecting the Moon's small obliquity, orbital eccentricity and inclination. Consider a particle moving in a periodic potential composed of degree two and three components, with the motion damped a constant frictional force. The angle $\phi = \theta - nt$ is the angle of the Earth as measured from the Moon, θ is the orientation angle of the Moon in inertial space, n is the orbital angular frequency, and t is time. The kinetic energy $T = \frac{1}{2}C\dot{\theta}^2$ and potential $U = -\frac{1}{2}\omega_0^2 \cos(2\phi) - \frac{1}{3}\epsilon\omega_0^2 \cos(3\phi + \phi_0)$ govern the conservative dynamics. Including the dissipative tidal torque, the equation of motion reads [1]:

$$\ddot{\phi} + \omega_0^2 \sin(2\phi) + \epsilon\omega_0^2 \sin(3\phi + \phi_0) = -\tau \mathrm{sign}(\dot{\phi}), \quad (1)$$

where the amplitudes of the degree 2 and 3 components are $\omega_0^2 = n\sqrt{3(B-A)/C}$ and $\epsilon\omega_0^2$ respectively, the phase difference between them is ϕ_0, and the dissipation is due to a constant torque τ acting in a direction opposite to that of $\dot{\phi}$. A, B, and C, and the equatorial and polar moments of inertia, respectively. An arbitrary mass configuration would have an additional $\sin\phi$ term arising from the degree 3 order 1 moment, but it is not needed to capture the essential dynamical behavior. The particle's potential energy oscillates while its total energy

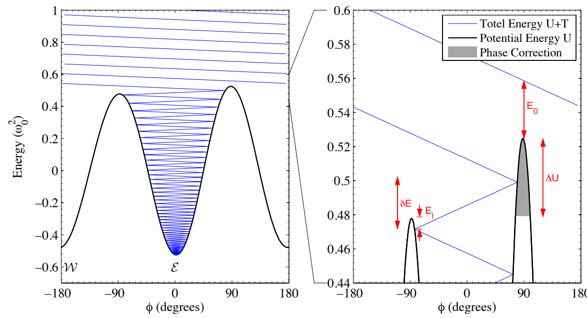

Figure 1: Representation of the total energy time evolution (blue), and the configuration potential (black), for a case where $\delta E/\Delta U = 0.65$. In the magnified plot on the right, the relevant energies for locking are indicated.

decays at a rate $-\tau|\dot{\phi}|$. For $\epsilon \ll 1$, the two minima and two maxima of the degree 2 potential are perturbed such that the energy difference between the minima is $\delta U = \frac{2}{3}\epsilon\omega_0^2 \cos\phi_0$ and between the maxima $\Delta U = \frac{2}{3}\epsilon\omega_0^2 \sin\phi_0$. The Moon's spin undergoes an analogous motion to that of this particle.

An example solution to Equation 1 obtained by numerical integration is shown in Figure 1. The initial energy dissipates at a rate $\tau\dot{\phi}$, or $2\pi\tau$ per cycle. When the total energy drops below the maximum potential energy the direction of motion reverses and the particle continues to oscillate and lose an energy of $\delta E \approx \pi\tau$ between stopping points separated by approximately π radians. Eventually the energy drops below the lower of the two potential maxima, and the particle falls into one of the two potential minima. The choice between the two minima is governed by the energy difference between the maxima, ΔU, and the energy dissipated per cycle between stops, δE.

We follow this simple dynamical system by direct integration of the trajectories, choosing parameters which set the difference in the potential energy maxima ΔU. We vary the dissipation parameter τ, thus varying the energy dissipated per cycle δE. For each resulting ratio of $\delta E/\Delta U$ we run 1000 simulations, randomizing the initial angle and kinetic energy, always above the energy required for complete circulation. We call the minimum preceding (East of) the higher potential maximum $\mathcal{E}$ and the minimum West of the maximum $\mathcal{W}$. The resulting probability of capture into the East minimum, $P(\mathcal{E})$, is plotted in Figure 2. Several features are noteworthy. When the energy dissipation, δE, and potential maxima asymmetry, ΔU, are equal, the probability $P(\mathcal{E}) = 1$. This may be understood as follows. The particle passes the highest potential maximum for the last time before stopping with an energy sufficient to carry out less than one, but more than one-half, cycle. It therefore captures into the adjacent potential minimum ($\mathcal{E}$), which with our choice of potential, is the higher of the two minima. The additional peaks in $P(\mathcal{E})$ similarly correspond

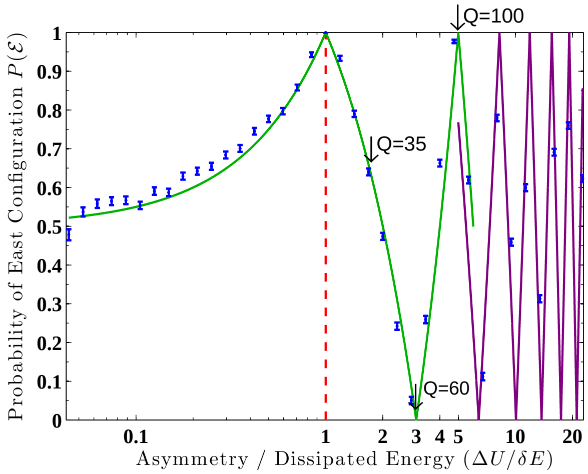

Figure 2: The probability of the $\mathcal{E}$ configuration obtained from sets of 1000 monte-carlo simulations with numerical integration of the synchronization process (points with error bars), a simple analytical solution for knife-edge potential (green line), and a correction to that solution appropriate for small dissipation (purple line). For concreteness we choose values describing the potential of $\omega_0^2 = 1$, $\epsilon = 0.1$, and $\phi_0 = \pi/4$.

to ratios of $\delta E/\Delta U$ that correspond to arriving at the potential maximum with kinetic energy sufficient for increasing integral number of oscillations before the total energy drops below the lower maximum. In the limit of high dissipation, stopping occurs in less than half a revolution after the last passage above one of the maxima, and hence the probability of capture into each of the minima approaches 1/2. At the other extreme, of low dissipation relative to the asymmetry energy, the probability function, $P(\mathcal{E})$, oscillates between 0 and 1, with an average of 1/2. As this discussion illustrates, the relevant energies are that of dissipation per cycle, and the difference in the potential maxima, not the difference in the potential minima.

An analytical solution is possible by approximating the shape of the potential with a correction for its width near the peak. The uncorrected and corrected analytical solutions are also shown in Figure 2.

Discussion

The torque, τ, which despins the satellite, may be obtained [1, 2] by considering the Moon's response to the degree two tidal potential parametrized by the Love number k_2 and quality factor Q. Gravity measurements indicate [3] $k_2 = 0.026 \pm 0.003$, whereas Q is constrained by lunar laser ranging [4] to be ~ 35. Assuming these values, the energy dissipated per cycle due to the integrated torque is $\delta E \sim 4.2 \times 10^{16}$ J, similar in magnitude to the asymmetry energy $\Delta U \sim 7.2 \times 10^{16}$ J. The two energies are comparable, resulting in interesting locking dynamics; the Moon despins into the current configuration with a mild preference, at a probability of $\sim 63\%$. If, however, $Q = 100$ in analogy with Earth's solid body, the preference is much stronger, with a probability for the current near-side of $\sim 97\%$. A value of $Q \sim 60$ is incompatible with locking in the current state.

The early Moon was hotter, closer to Earth, and its mass distribution could have been different from today's. With a residue of accretional heat and a higher rate of tidal dissipation at a closer distance, the Moon's mantle convection would have been more vigorous and could have dynamically supported larger geoid anomalies. The relative timing of maria emplacement and locking into resonance is not clear. [*e,g, 5, 6]*. Dissipation in the past depends upon the orbital frequency and Q, which are difficult to constrain. The dissipation energy δE scales as $\sim \frac{k_2}{Q}\frac{R_M^5}{a^6}GM_E^2$ where M_E is the Earth's mass. The energy difference between the potential minima and maxima arises dominantly due to the octupole gravity moment of the Moon. This quantity scales with $C_{3m}M_M M_E R_M^3/a^4$, where M_m is the lunar mass and C_{3m} denotes the degree 3 gravity coefficients. Hence, in a smaller orbit, the dissipative energy term was larger and may have been comparable to, or even exceeded the asymmetry energy.

Multiple impact craters resulting from bodies capable of knocking the Moon out of synchronous rotation are recorded in the lunar crust, suggesting the Moon was kicked out of resonance and relocked after its formation [7, 8]. It is difficult to identify or date the youngest such event, although it appears the majority occurred more than 3.8 billion years [8, 9]. A statistical estimate of the clustering of ancient large craters indicates that the current trailing hemisphere might have been the leading one during the time the oldest subset of these large basins formed [8]. These basins occur early enough in lunar history that using present day parameters for the capture dynamics is not justified. However, in the event that the last such reorienting impact occurred sufficiently late in lunar history such that the Moon's gravity, orbital period, and Love number were similar to today's, then the estimates of Q given above are relevant.

We show that for the current Moon, the asymmetry and dissipative energies are close to each other, resulting in the interesting capture dynamics. This, rather than the semblance of a man's face on the near-side, may be the surprising coincidence of our Moon.

Acknowledgments: We would like to thank Bruce Bills and Marc Wieczorek for enjoyable discussions.

References

[1] P. Goldreich & S. Peale. *Astron. J.*, *71*, 425, 1966.

[2] W. M. Kaula. *Reviews of Geophysics and Space Physics*, *2*, 661–685, 1964.

[3] A. S. Konopliv, et al. *Icarus*, *150*, 1–18, 2001.

[4] J. G. Williams & D. H. Boggs. *Proceedings of the 16th International Workshop on Laser Ranging*, 2008.

[5] D. E. Loper & C. L. Werner. *J. Geophys. Res. Planets*, *107*, 131–137, 2002.

[6] M. Jutzi & E. Asphaug. *Nature*, *476*, 69–72, 2011.

[7] H. J. Melosh. *Earth and Planetary Science Letters*, *26*, 353–360, 1975.

[8] M. A. Wieczorek & M. Le Feuvre. *Icarus*, *200*, 358–366, 2009.

[9] D. Stöffler & G. Ryder. *Space Sci. Rev.*, *96*, 9–54, 2001.