PRIMARY ACCRETION BY TURBULENT CONCENTRATION: THE RATE OF PLANETESIMAL FORMATION AND THE ROLE OF VORTEX TUBES. Jeffrey N. Cuzzi[1] and Robert Hogan[2], [1]*Ames Research Center, NASA, MS 245-3, Moffett Field, CA, USA (Jeffrey.Cuzzi@nasa.gov);* [2]*BAERI inc.; Sonoma, CA.*

Introduction: The initial accretion of primitive bodies (asteroids, KBOs, etc) from freely-floating nebula particles remains problematic. Traditional growth-by-sticking models encounter a formidable “meter-size barrier" (or even a cm-size barrier) in turbulent nebulae, but nonturbulent nebulae form large asteroids too quickly to explain long spreads in formation times, or the dearth of melted asteroids [1,2]. Another clue is an apparent peak in the pre-depletion, pre-erosion mass distribution of asteroids [3-6]. A recently advanced scenario has potential to naturally explain the characteristic chondrule-dominated fabric of most primitive meteorites as well as the overall properties of their parent bodies [4,7]. In this scenario, dense clumps of size-sorted particles, formed in turbulence, can under certain conditions shrink inexorably on 100-1000 orbit timescales and form sandpile planetesimals. Other models focus on the role of much larger (boulder-size) components, (most recently [8]), but we will focus on chondrule-size particles as independent constituents. Here we show that the rate of planetesimal formation was probably underestimated in [4] and overestimated in [7], and discuss the implications. We then remark on how *vortex tubes* in turbulence may help explain why such small particles can be affected on length-scales nominally ranging from the smallest to the largest, a key element of the models of [4] and [7].

Cascade model and thresholds: We define Φ as the local ratio of particle mass density ρ_p to gas mass density ρ_g, and ω a local vorticity or eddy frequency in nebula turbulence. The outcome of turbulent concentration can be captured statistically by a *cascade model* [4,7,9] which predicts probability distribution functions (PDFs) $P(\Phi,\omega)$ for dense particle clumps; these PDFs are essentially volume fractions in the nebula having the given properties. A cascade model presumes that, as energy flows from large eddies to smaller ones, particles and fluid properties are partitioned unequally from “parent” into “daughter” eddies. To date, the partition coefficients or *multipliers* have been obtained from 3D numerical simulations at small scales where good statistics can be realized [9]. $P(\Phi,\omega)$ is a function of lengthscale, itself characterized by *level* N in the turbulent cascade, and the volume fraction $P(\Phi,\omega)$ of more extreme values of Φ and ω increases at deeper levels [9,4,7]. Each cascade level is associated with a nebula lengthscale $l = 2^{-N/3}L$ where $L = H\alpha^{1/2}$ is the largest eddy scale, H is the gas vertical scale height, and α the nebula turbulent viscosity parameter [4]. The largest eddy has a characteristic timescale t_L, on the order of the orbit time, and smaller eddies have shorter timescales $t_l \sim t_L(l/L)^{2/3} = 2^{-2N/9}t_L$ [4,7].

Dense particle clumps can be stabilized against disruption while they slowly contract under self-gravity into sandpile planetesimals only if they exceed certain thresholds of Φ, ω, and l [4,7]. At any given snapshot in time, the available amount of *surface mass density* σ_{av} which exceeds this threshold and starts to form planetesimals is given by the PDF (or volume fraction) exceeding the stability thresholds, multiplied by the mass of the solids within this volume. This can be written roughly as $\sigma_{av} = f\Sigma_g\Phi^*P^*$, where Σ_g is the nebula gas surface density, f represents a fraction lying close enough to

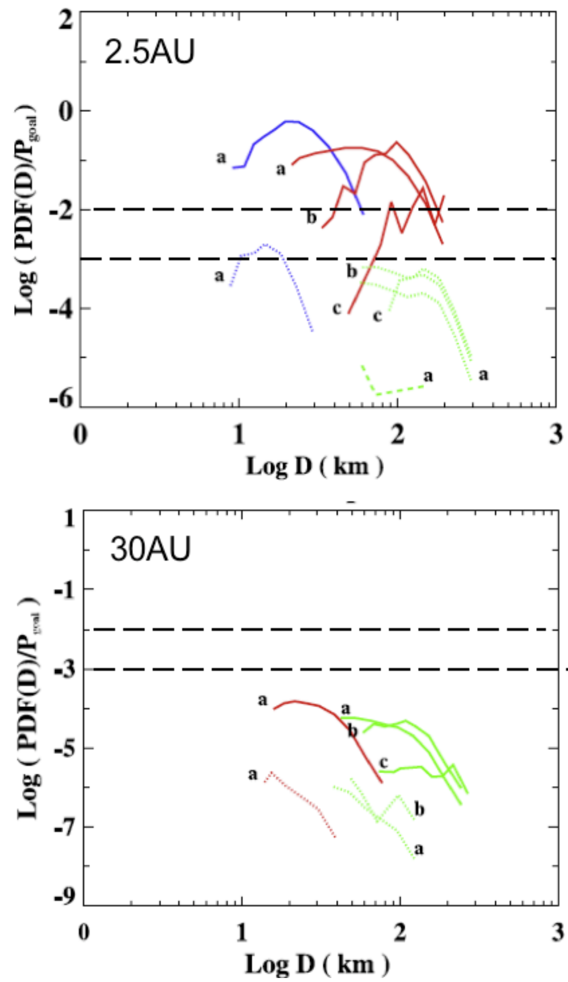

Figure 1: Plots of the predicted Initial Mass Function (IMF) for primary planetesimals at 2.5AU (with 10x enhancements of solids near the midplane) and 30 AU (with no enhancement), for one specific nebula model [4]. The various symbols and colors represent different nebula parameter sets. Note the well-defined mode at 10’s - 100’s of km diameter of the final body. The vertical axis is labeled by the log of the ratio of the model PDF to a “goal” value derived from the imposed planetesimal formation rate $\dot{\sigma}_{pa}$ *assuming* $t_{pa} = t_{sed} \sim$ *100-1000 orbits* (see text); curves with peaks near unity on this axis were thus deemed promising by [4] (flatter disk density profiles did better at 30AU). The dashed horizontal lines indicate where PDF(D)=P_{goal} would lie if $t_{pa} = t_L$ instead of t_{sed}. A much wider and more plausible range of nebula properties can now satisfy the stipulated mass production rate at 2.5AU, and the outer nebula situation beomes achievable with only a modest increase in the solids/gas ratio near the midplane. The model predictions of [4] also become more robust to uncertainties in the parameters of the cascade.

the midplane for vertical settling not to be a concern, Φ^* is a typical solids/gas ratio for incipient primary accretion, and P^* is its PDF or volume fraction. This quantity is essentially the numerator of equation 9 of [4] and equation 14 of [7]. Both studies proceed to get the primary accretion planetesimal formation *rate* $\dot{\sigma}_{pa}$ dividing by some timescale we call t_{pa}, and this is where a discrepancy arises in the conclusions.

The meaning of t_{pa} is that time in which an independent set of planetesimals is initiated. Cuzzi et al [4] conservatively adopted the sedimentation time of the clump, $t_{pa} = t_{sed} \sim 100-1000t_L$, or the time it takes for sufficiently dense clumps to settle into sandpile planetesimals. Chambers [7] assumed t_{pa} was on the order of the characteristic time *at the lengthscale l of the initial clump*, $t_{pa} = t_l << t_L$ (his eqn. 4). Both papers set roughly the same requirement of producing several $M_\oplus$ of planetesimals in the asteroid belt region, and tens of $M_\oplus$ in the Kuiper belt, in a few Myr. While the two papers present their results differently, [4] required slightly higher gas and solid densities, lower nebula α, and/or and smaller radial pressure gradients than [7] to satisfy the stipulated $\dot{M}_{pa}$.

We now believe that $t_{pa} \sim t_L$, lying between the assumptions of [4] and [7]. Another way to think of t_{pa} is that timescale on which a physically and statistically independent realization of the particle and fluid velocity and density fields is manifested. This gives the rate at which the mass density $\sigma_{av} = f\Sigma_g\Phi^*P^*$ can be transformed into planetesimals. In retrospect it is clear that t_{sed} is not correct because, once a clump has started down this path, it inevitably becomes a planetesimal, so it is the *appearance* rate of the *suitably dense clumps* that matters. It is well-known that the *fluid* field refreshes on the timescale t_L; several particle/fluid dynamical studies show that the particle density field represented by $P(\Phi, \omega)$ and P^* is also independently refreshed in t_L [10,11]. The effects of this change are shown in the figure and discussed more in the caption. The greater robustness of the scenario (relative to the predictions in [4] at least), also gives it a greater resilience to anticipated uncertainties in key parameters, such as the cascade model prediction of $P(\Phi, \omega)$ (see below). On the other hand, the $\dot{\sigma}_{av}$ of [7] (eqn. 14) *decreases* by a factor of $t_l/t_L \sim 2^{-2N/9}$ ~60 for N=20, requiring either a decrease in α or the pressure gradient parameter, or an increase in the gas and/or solids density (from, *eg.*, figure 1). Thus the somewhat disparate predictions of [4] and [7], based on essentially the same physics, become more convergent.

Multipliers, scale independence, and vortex tubes: The cascade model is based on scale invariance of the multipliers, which current numerical models can now only determine with accuracy at scales near the Kolmogorov or dissipation scale. It remains unproven whether multipliers determined at such small scales are valid at large scales. Physically, one can wonder why large-scale eddies, with nominally *slow* overturn times, should affect particles like chondrules, with stopping times close to the smallest eddy time, in the same way as we measure at *small* scales where the eddies *do* have these timescales. These uncertainties could be important, as we have noted in the past and as has been pointed out recently [12]. However, there is reason to believe that this concern is not as problematic as it seems.

Classical thinking about turbulence (the Kolmogorov 1941 theory and the so-called Richardson cascade of large equidimensional eddies to smaller, but also equidimensional eddies) is now known to overlook a key aspect of turbulence - elongated, coherent structures or *vortex tubes* containing a substantial fraction of the vorticity and dissipation [13-16]. These vortex tubes are created when eddies are stretched along their rotation axis by nearby straining flows, and are stable for times of t_L or longer [13-16]. While these structures have a distribution of length and timescales, with axial scale up to the integral scale, their dominant diameters and eddy times are roughly the Kolmogorov scale and time [13-16]. Their "spinning" nature, which persists for many of their own eddy times, allows them to affect particles with Kolmogorov-like stopping times over lengthscales up to the integral scale, at least along their long dimension and apparently even perpendicular to it [17,18]. That is, if vortex tubes with Kolmogorov radii and timescales (but a range of axial lengthscales) dominate vorticity, the same multipliers we measure at small scales might indeed apply at large scales. *Dissipation* of turbulent kinetic energy by viscosity which, like particle concentration, correlates with velocity shear, has long been known to have level-independent multiplier functions over a very wide range [19]. Indeed [9; their fig. 4] showed that multipliers determined on small scales and applied in cascades of different depths reproduce actual 3D properties of $P(\Phi, \omega)$ quite well, even if at somewhat higher PDF range (10^{-3}) than of interest for our application (10^{-6}); thus further refinement along these lines is very important. While some studies have shown that the optimally concentrated particles might be somewhat larger at larger spatial scales [12,20], this effect may only modify the value of the gas density/particle stopping time combination that is optimal for concentration by factors of a few.

References: [1] Cuzzi J.N. and Weidenschilling S.J. (2006) in "Meteorites and the Early Solar System II, Univ. Arizona Press. [2] Zsom A. et al 2010, A&A 513, id.A57; [3] Bottke W. et al 2005, Icarus 175, 111; [4] Cuzzi J.N., R.C. Hogan, and W.F. Bottke 2010 Icarus, 208, 518 [5] Morbidelli A. et al 2009, Icarus 204, 558; [6] Weidenschilling S. 2011 Icarus, 214, 671 [7] Chambers, J. 2010; Icarus 208, 505 [8] Johansen, A. et al (2011) Workshop on Formation of the First Solids in the Solar System, Kauai, Hawaii. LPI Contr. 1639; [9] Hogan R. and J. Cuzzi (2007) Phys Rev E 75, 056305; [10] Reutsh G.R. and M. Maxey 1992, Phys Fluids A, 2747; [11] Cencini M. et al 2006, J. Turb. 7, #1468-5248; [12] Pan, L. et al, ApJ 740, article id. 6 (2011); [13] Jimenez, J. et al 1993, J. Fluid Mech 255, 65; [14] Moffatt H.K. et al. 1994, J. Fluid Mech. 259, 241; [15] Pirozzoli, S. 2012, Physica D, 241, 202; [16] Vincent A. and M. Menneguzzi 1994, J. Fluid Mech. 258, 245; [17] [17] Ayyalasomayajula et al 2008, Phys. Fluids 20, 095104; [18] Calzavarini E. et al 2008; J. Fluid Mech. 607, 13; [19] Sreenivasan and Stolovitsky 1995, J. Stat. Phys. 78, 311; [20] Bec J. et al. 2007, Phys. Rev. Lett. 98, 084502; We thank NASA's Origins of Solar Systems program for supporting this work.