INTRODUCTION: Basic observations have long suggested that there is a relationship between lava flow dimensions and composition, with mafic flows being generally longer and thinner than felsic flows. Furthermore, observations of active flows, especially in Hawaii and Mt. Etna, found an empirical link between effusion rate and flow length [1] or flow area [2]. Because simple Newtonian liquids do not stop on slopes the way lava flows do, the relationship between lava composition (via rheology) has often been simulated using a Bingham plastic fluid model which includes three simple equations [3].

\[ \sigma = \rho g H^2/W \]  
\[ \sigma = \rho g H \alpha \]  
\[ \sigma = 2 \rho g W_b \alpha^2 \]  

where \( \sigma \) is yield strength, \( \rho \) is density, \( g \) is gravitational acceleration, \( H \) is flow thickness, \( W \) is flow width, \( \alpha \) is slope, and \( W_b \) is the width of channel levees. Similarly, the Gratz model is one of the simplest formulas useful in industrial applications for freezing flows and links effusion rate to flow length [4].

\[ Gz = Q H/(\kappa L W) \]  

where \( Gz \) is the Gratz number with lavas stopping at a value of about 300, \( Q \) is effusion rate, \( \kappa \) is thermal diffusivity, and \( L \) is the length of the flow. Kilburn and Lopes [5] produced a similar relationship for simple aa lobes.

\[ \tau = W H^2 \alpha/\kappa L c \]  

where \( \tau \) is the eruption duration and \( c \) is a constant with a value of 3 for most flows.

These and other similar simple relationships are especially appealing to planetary scientists where most flows can only be studied via remote sensing long after they have been emplaced. However, as detailed below, there are significant limitations for each of these expressions. In the following, the goal is to derive similarly simple expressions for planetary volcanologists. As such, the emphasis is on mafic flows and deriving parameters such as lava rheology that are difficult to measure for planetary flows from parameters such as flow dimensions that are relatively easy to measure.

WHY THE MODELS ARE WRONG: In the case of the Bingham model, it assumes that lava stops wherever it is too thin to overcome the lava’s inherent yield strength. Therefore the mobile interior of lava channels should stand above the channel levees. However, for mafic flows, this only occurs in very rare aa flows. Levees are actually constructed from successive overflows, much like levees on a river. At the flow front, the Bingham model does not allow for the thin breakouts from the front of the flow that are typical of both aa and pahoehoe flows as they advance. Therefore, the Bingham model does not have a physical basis for providing meaningful results for many types of flows, especially pahoeho lavas. This means that the yield strength that is calculated from (1a)-(1c) is totally unrelated to the intrinsic yield strength of the lava as measured in the laboratory and the three formulations rarely give consistent results.

The Gratz method was developed for conductively cooling pipe flow, so it is most applicable to lava tubes. However it has often been used for lava channels with significant heat loss via thermal radiation where it is not appropriate. Equation (3) from [5] is intended only for simple aa flows. These simple expressions may have more validity when applied to relatively small and simple felsic lavas that are not typically of interest in planetary volcanology. And yet their continuing use indicates that there still is some value to the numbers that come out of these models. In order to revisit these incorrect but apparently useful models, we need to examine lava flows at their most basic.

Basic Principle: The basic method followed here is similar to that of many previous studies [e.g., 4-6]. The simplest way to look at a frozen lava flow is that it records when the timescale for the solidification of the flow matches the timescale for the advance of the flow. If we can properly parametrize these two timescales, we are well on our way to a simple and valid model for how lava flows work. While the goal is to be as simple as possible, we cannot avoid classifying lava flows into at least three broad categories: (1) open flows, (2) insulated flows with mobile upper crusts, and (3) insulated flows with non-translating crusts. Channelized aa and pahohoe flows fall in the first category, as do open sheet flows which are usually only found near vents. Rubbly pahohoe flows, equivalent to platy-ridged lava first identified on Mars, typifies the second group but some aa flows also belong here. Lava tubes and inflated sheet flows both fall in the third class. Pahoehoe more commonly behaves in this manner, but there are tube-fed aa flows and inflated aa sheets.

The Flow Cooling Timescale: It is simplest to think of the cooling timescale as the time required to cool the entire flow such that it crystallizes too much to flow. ~50 °C has been used as a canonical value for the cooling required to stop a flow, but the exact value is not important for the following discussion. All that is needed is that the required cooling be broadly sim-
lar for the flows being analyzed. In equation form, we are looking for
\[ \tau_c = \left( \frac{H \Delta T_c \rho C_p^*}{\Phi} \right) \]
where \( \tau_c \) is the cooling timescale, \( \Delta T_c \) is the cooling required to stop the flow, \( C_p^* \) is heat capacity taking into account latent heat, and \( \Phi \) is the heat loss. So what controls \( \Phi \)? For open flows the majority of the heat loss is via thermal radiation from breaks and cracks in the crust [7]. This can be written as
\[ \Phi = (f \sigma (T_i^4 - T_a^4)) + (1-f)(T_c^4 - T_a^4) \]
where \( f \) is the crack fraction, \( \sigma \) is the Stefan Boltzmann constant, \( T_i \) is the temperature of the interior of the flow, \( T_a \) is the ambient temperature, and \( T_c \) is the crust temperature. If \( f \) is even a fraction of a percent and \( T_c \) is significantly less than \( T_i \), this simplifies to
\[ \Phi = f \sigma T_i^4 \]
Lavas with higher \( T_i \) tend to have higher \( \rho \) and \( C_p^* \) and is partially compensated for natural variations in \( T_i \). \( f \) is thus the single most influential parameter on \( \Phi \) for open flows. It should be noted that lava crust entrainment can be a very important element in the thermal budget for open flows [7]. However, the crust that becomes entrained must first form via cooling of the lava surface. Thus, in the overall picture, the effect of entrainment can be captured by using an appropriately increased value for \( f \) that recognizes this effect.

For all types of insulated flows, heat loss is via conduction through the crust
\[ \Phi = k \left( \frac{T_i - T_a}{H_c} \right) \]  
where \( k \) is thermal conductivity and \( H_c \) is the thickness of the upper crust. To first order, \( k \), \( T_i \), and \( T_a \) are similar for mafic flows, so \( H_c \) is the key variable for these flows. The way \( H_c \) changes with time can be quite different for stationary and mobile upper crusts. For stationary crusts, \( H_c \) grows proportional to the square root of time [8]. However rubbly pahoehoe flows, at least sometimes, form a thick crust quite suddenly as a surge of liquid lava passes within the flow [9].

### The Flow Advance Timescale: A Similar process can be applied to \( \tau_f \), the flow advance timescale.
\[ \tau_f = \frac{V - \bar{Q}}{W \cdot L \cdot H / Q} \]
where \( V \) is the flow volume. While effusion rate (Q) is an interesting parameter on its own, it is useful to substitute \( V / WH \) for Q where \( \bar{Q} \) is the mean flow velocity. This formulation uses H synonymously for both the flow thickness and the thickness of the flowing part of the flow. These two thicknesses are often very similar except for long-lived insulated flows with stationary crusts. In those cases \( H_a \), the thickness of the mobile interior can be \( 1/2 \) of \( H \), or less [8]. To first order, the moving interior of a mafic lava flow can be modeled as a Newtonian fluid. In this case
\[ \bar{Q} = \rho \cdot g \cdot \alpha \cdot H^2 / C \eta \]
where \( \eta \) is viscosity, \( C \) is a constant with the value of 3 for a moving flow top, 8 for a filled tube, and 12 for a sheet flow with a stationary crust. Because the actual rheology of lavas is complex, best described as a visco-elastic fluid with strain-rate, temperature, crystal, and bubble dependent properties, \( \eta \) should probably be considered a broad “fluidity” rather than a formal viscosity. Still, the values of \( \eta \) should be comparable with laboratory measurements of the rheology of lavas.

Again, for insulated flows with stationary crusts \( H \) is the thickness of the liquid part of the flow, not the entire flow thickness. Substituting into (7) gives us
\[ \tau_f = \frac{V}{W \cdot L \cdot \eta \cdot C \cdot \rho \cdot g \cdot \alpha \cdot H^2} \]

### The Combined Model: By setting \( \tau_f \) and \( \tau_c \) to be equal to each other in equations (4) and (7)
\[ \Delta T_c \cdot \rho \cdot C_p^* \cdot \bar{Q} = \Phi \cdot W \cdot L \]
which can be rearranged such that
\[ \bar{Q} = \Phi \cdot W \cdot L / \Delta T_c \cdot \rho \cdot C_p^* \]
For flows within a given flow category with similar thermal properties (\( \Delta T_c \), \( \rho \) and \( C_p^* \)) this establishes why there is a strong relationship between \( \bar{Q} \) and flow area (\( W \cdot L \)) [2]. The discrepancies should primarily be related to variations in heat loss (\( \Phi \)) with \( f \) and \( H_c \) being the most important variables for open and insulated flows, respectively.

Using (9) instead of (7) one derives
\[ H \cdot \Delta T_c \cdot \rho \cdot C_p^* \cdot g \cdot \alpha \cdot H^2 = \Phi \cdot L \cdot \eta \cdot C \]

For mafic flows, \( \Delta T_c \), \( \rho \) and \( C_p^* \) will be similar and \( g \) and \( \alpha \) can be well-constrained. Within a given category of lava flow, \( \Phi \) and \( C \) should also be similar. Thus (10b) can be simplified to
\[ \eta = g \cdot \alpha \cdot H^2 \cdot C / L \]
where \( C \) is roughly constant for each of the 3 major categories of flows but is expected to be very different for the different categories. Simply measuring \( H^2 / L \), and compensating for \( g \) and \( \alpha \), we can estimate the fluidity of the flow. Note that this extremely simple relationship has many of the same terms as (1)-(3) but is not identical. Further comparison to field data is required, but (13) better matches the basic observation that it is the thickness of the flow that is most diagnostic of rheology and thus composition.

### References: