DYNAMICAL WEAK STATISTICAL EQUILIBRIUM AND THE NEUTRON-RICH IRON-GROUP ISOTOPES.
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Introduction: Calcium-48 and other neutron-rich iron-group isotopes such as ^{50}Ti and ^{54}Cr show roughly correlated excesses and deficits in FUN CAIs and primitive hibonite grains (e.g., [1]). Such correlations in these primitive samples have important implications for our understanding of the early Solar System.

These isotopes also have considerable interest for nucleosynthesis theory. They have substantial production in expansions of low-entropy matter [3], an environment that most naturally occurs in thermonuclear explosions of white dwarf stars (Type Ia supernovae) when the density is great enough that electron capture reactions can occur to drive the matter sufficiently neutron rich [4].

The thermonuclear supernovae that produce the neutron-rich isotopes are only a small fraction of all such Ia events. The reason is that the density must get high enough to allow nuclei to capture enough electrons to drive matter to the electron-to-nucleon ratio Y_e appropriate for ^{48}Ca ($Y_e = 0.417$). Once the density is sufficiently high, however, the electrons can come into a balance with other weak reactions, which sets the possible Y_e in the matter.

If neutrinos are trapped in the matter, this is normal weak statistical equilibrium, a true free energy minimum with the simple constraint that the total number of nucleons is fixed. If neutrinos may freely stream out of the matter, however, this is dynamical weak statistical equilibrium, which is a free energy minimum with the constraint that the rate of change of Y_e is zero [4]. In this abstract, we study these equilibria and their implications for the neutron-rich iron-group isotopes.

Weak Statistical Equilibrium: Nuclear statistical equilibrium (NSE) is the condition that all strong and electromagnetic reactions are in balance in a nucleosynthetic environment. In such a case, the chemical potentials of free neutrons μ_n, protons μ_p, and a heavy species i with atomic number Z_i and mass number A_i $\mu(Z_i, A_i)$ are related by

$$Z_i\mu_p + (A_i - Z_i)\mu_n = \mu(Z_i, A_i) \quad (1)$$

This equation shows that there is no thermodynamic cost in assembling Z free protons and N = A-Z free neutrons into a nucleus (Z,A). If the weak reactions, which convert neutrons into protons and *vice versa*, also are in equilibrium, then weak statistical equilibrium (WSE) applies, in which case

$$\mu_p + \mu_e = \mu_n + \mu_\nu \quad (2)$$

This equation shows that there is no cost in converting a neutron and neutrino into an electron and a proton, and back. This form of weak statistical equilibrium holds if the neutrinos are trapped. If the neutrinos can escape, however, which is certainly true below a density of ~10^{12} g/cc, the appropriate equilibrium is not that characterized by equation (2). Rather, it is an NSE such that the electron capture and beta-plus decays of all the nuclei balance the positron capture and beta-minus decays; thus,

$$\sum_i \left(\Lambda_i^{bm} + \Lambda_i^{pc} - \Lambda_i^{bp} - \Lambda_i^{ec}\right)Y_i \equiv \sum_i \Lambda_i Y_i = 0 \quad (3)$$

where Λ represents a weak reaction rate per nucleus, and *bm, pc, bp,* and *ec* represent beta-minus, positron-capture, beta-plus, and electron-capture rates, respectively. Y_i is the abundance of species i. Operationally, to compute this equilibrium, one adjusts Y_e until the NSE satisfies equation (3).

We have applied the Lagrange-multiplier technique of [4] to show the that dynamical weak statistical equilibrium condition can be represented as

$$\mu(Z_i, A_i) = Z_i\mu_p + (A_i - Z_i)\mu_n + \lambda(\Lambda_i - Z_i\Lambda_p - (A_i - Z_i)\Lambda_n) \quad (4)$$

where λ is a Lagrange multiplier and Λ_p and Λ_n, are weak reaction rates on free protons and neutrons, respectively. This equation shows that the normal NSE condition on a heavy species is modified by the total weak rate on that species relative to the net weak rate on the individual nucleons making the species up. We have also been able to show that the condition on the free nucleons is

$$\mu_p + \mu_e = \mu_n + \lambda\left[\left(\frac{\partial \dot{Y}_e}{\partial Y_e}\right)_{free} - \left(\frac{\partial \dot{Y}_e}{\partial Y_e}\right)\right] \qquad (5)$$

where $\dot{Y}_e$ is the rate of change of Y_e with respect to time and *free* refers to the case that all nucleons are free. By comparison with equation (2), we see that the last term in equation (5) is an effective neutrino chemical potential. Thus, the effective neutrino chemical potential is related to how the time rate of change of the electron fraction changes with the electron fraction compared to the case where all the nucleons are free.

Calculations: We have written a set of tools to compute weak interaction rates and statistical equilibria called NucNet Tools [5]. Since a full set of microscopic calculations of weak interaction rates is not available, we have used developed routines in NucNet Tools to compute rates approximations of [4]. For other relevant data we use the nuclear data available from the Joint Institute for Nuclear Astrophysics (JINA) nuclide database [6].

Figure 1 shows the rate for the reaction $^{56}Fe + e^- \rightarrow {}^{56}Mn + \nu_e$ as a function of the product of the mass density times Y_e. The rate is computed at a fixed temperature of 8 billion Kelvins. As is apparent, as the density rises, the electron capture rate grows. Such electron capture drives the material neutron rich.

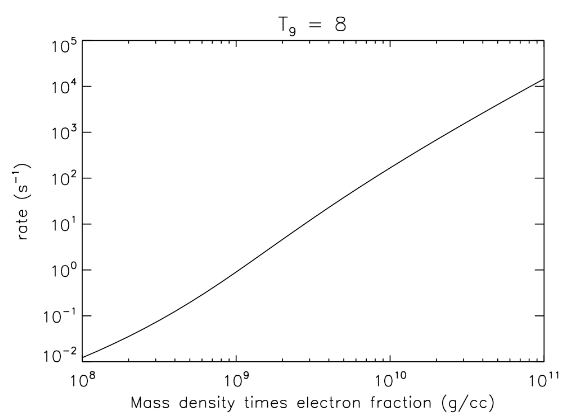

Figure 1. The rate for electron capture on ^{56}Fe as a function of mass density times Y_e as computed from routines in NucNet Tools [5] based on expressions in [4].

With the capture rates available, it is then possible to compute weak statistical equilibrium Y_e as a function of density. Figure 2 shows the equilibrium Y_e for dynamical WSE and for regular WSE but with a neutrino chemical potential divided by kT (where k is Boltzmann's constant) of 0. Since ^{48}Ca has a Y_e of 0.417, it is evident from Figure 2 that the white dwarf density must reach roughly 5-6 x 10^9 g/cc or higher to produce significant amounts of ^{48}Ca. This is in good agreement with simple Ia models [7] and more detailed calculations [3]. It is interesting that the equilibrium Y_e is lower in the dynamical WSE case than the regular WSE case with zero electron chemical potential. This result was noted in [4], and we see that the effective neutrino chemical potential in equation (5) is thus negative, at least under the conditions of interest to production of the neutron-rich iron-group isotopes.

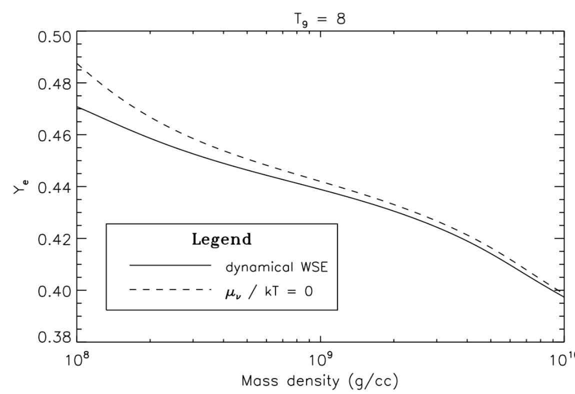

Figure 2. The weak-statistical equilibrium Y_e at 8 billion Kelvins for dynamical WSE and for WSE with trapped neutrinos with chemical potential zero.

Since all of the computational tools discussed in this abstract are available for download, we invite the interested reader to explore other conditions relevant to ^{48}Ca nucleosynthesis. The authors are writing tutorials and other guides to help others in this effort. These tutorials and guides will be available from the NucNet Tools web site and links therein.

References: [1] Meyer B. S. and Zinner E. in Meteorites and the Early Solar System II (Tucson: University of Arizona Press), p.69-108.[2] Meyer B. S., Krishnan T. D., and Clayton D. D. (1996) *Astrophys. J., 462,* 825-838. [3] Woosley, S. E. (1997) *Astrophys. J., 476,* 801-810. [4] Arcones A. et al (2010) *Astrophys.Ap., 522,* A25. [5] Available at http://sourceforge.net/projects/nucnet-tools [6] http://groups.nscl.msu.edu/jina/nucdatalib [7] Yu T. and Meyer B. S. (2012) this volume.