Effect of Stochastic Charging on Cosmic Dust Aggregation

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Introduction

Coagulation of cosmic dust grains is a fundamental process which takes place in environments such as presolar nebulae, circumstellar disks and protoplanetary disks. Cosmic dust grains can become charged through interaction with their plasma environment, and the resultant electrostatic force among dust grains can strongly affect their coagulation rate. Since ions and electrons are collected on the surface of the dust grain at random time intervals, the electrical charge of a dust grain experiences stochastic fluctuations. In this study, a set of stochastic differential equations is developed to model these fluctuations over the surface of an irregularly-shaped aggregate. The influence of the charge fluctuations on the coagulation process (driven by a turbulent relative velocity) and the physical characteristics of the aggregates formed is examined. It is shown that dust with small charges (due to the small size of the dust or a tenuous plasma environment) are affected most strongly.

Stochastic Charging of Monomers

Below left, the charging time history for a single spherical grain (a). The charge has a normal distribution about the mean (b). Below right, the charge and variance are shown for spheres as a function of radius for hydrogen plasma with $T = 900$ K and electron and ion densities (a) $n_i = n_e = 5 \times 10^7$ m$^{-3}$, (b) $n_i = 0.1 n_e$.

Stochastic Charging of Aggregates

Below left, the charge (a) and dipole moment (b) are compared for stochastic and non-stochastically charging conditions showing small difference. The variation in the charge (below right) can vary by almost an order of magnitude from that predicted by theory.

Collison Statistics

At larger $N$, stochastic charging increases the collision probability for the highly charged dust, but decreases that for the dust with small charges (left). The average size of an aggregate incorporated into a larger one tends to approach a constant size (middle) while the average size for the non-stochastically charged aggregates continues to increase. The small ($N \leq 20$) aggregates tend to be more efficient at colliding with the larger grains than do monomers (right).

Calculating Charge on Aggregates

Stochastic Fluctuations

A master equation describes the random charge fluctuations on the aggregates:

$$\frac{dZ_p(t)}{dt} = \sum_{i,j,k} I_{ij}(Z_e - e_i) F(Z_e - e_i) + I_{ik}(Z_e - e_k) F(Z_e - e_k) - \left[ I_{pi} + I_{pi}^d \right] Z_p(t).$$

$N$ is the total number of patches on the aggregate, $Z = \{Z_1, Z_2, ..., Z_N\} \in \mathbb{R}^N$ is the vector of the elementary charges collected on patches, $\mathbb{R}^N$ is the probability density function of a state at which the patch $p_i$ has $Z_i$ charges, etc., $I_{pi}$ and $I_{pi}^d$ are the currents of ions and electrons, respectively, to the patch $p_i$ and $e_i \in \mathbb{R}^N$ is the unit vector, e.g., $e_i = (0,0,1,0,0,...)$.

A Fokker-Planck equation may be obtained from the master equation which is statistically equivalent to a stochastic differential equation for the charge of each patch

$$\frac{dZ_p(t)}{dt} = \left( I_{pi}(Z_e - e_i) F(Z_e - e_i) + I_{pi}^d(Z_e - e_i) F(Z_e - e_i) - \left[ I_{pi} + I_{pi}^d \right] Z_p(t) \right) dW(t),$$

where $dW(t)$ is a Wiener process. This equation is solved numerically by the Euler-Maruyama method:

$$Z_p(t + \Delta t) = Z_p(t) + I_{pi}(Z_e - e_i) F(Z_e - e_i) \Delta t + I_{pi}^d(Z_e - e_i) F(Z_e - e_i) \frac{dW(t)}{\Delta t},$$

where $\Delta t$ is the time step, $Z_p(t)$ is the charge of the patch $p$ at time step $n$ and $I_{pi}$ is a random number with a normal distribution. $Z_p(t)$ is calculated using orbital motion limited theory (OML) with a line of sight (LOS) approximation.

Aggregate Characteristics

The morphology of the aggregates influences their coupling to the gas and resultant dynamics.

Compaction Factor

Fluffier aggregates, characterized by lower compactness factors, are more strongly coupled to the gas and have greater collision cross sections.

OML_LOS

Divide aggregate surface into many patches

• Determine open lines of sight to points at center of each patch (LOS_factor)

• Calculate current densities to each point, $J_i(t)$ and $J_d(t)$, using LOS_factor and potential $\phi$ due to charge on each patch

• Calculate charge collected on each patch with area $A_i$ during time interval $dt$: $Q_i = \sum J_i A_i dt$

• Update charging currents and iterate in time

A 2D representation of the open lines of sight to three points on the surface of an aggregate.

References


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