Tidal Response of a Laterally Varying Moon: an Application of Perturbation Theory

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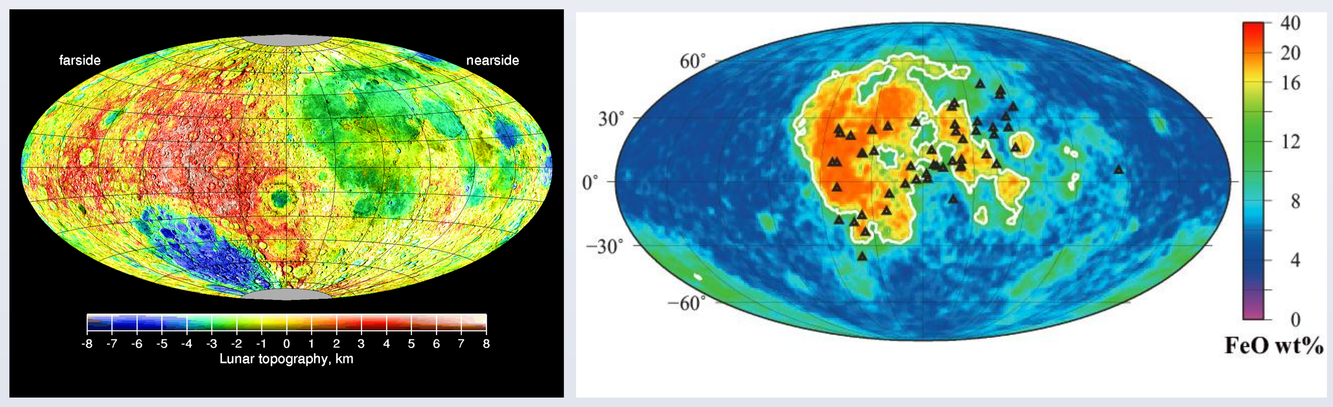
INTRODUCTION

The Moon's hemispherical asymmetries:

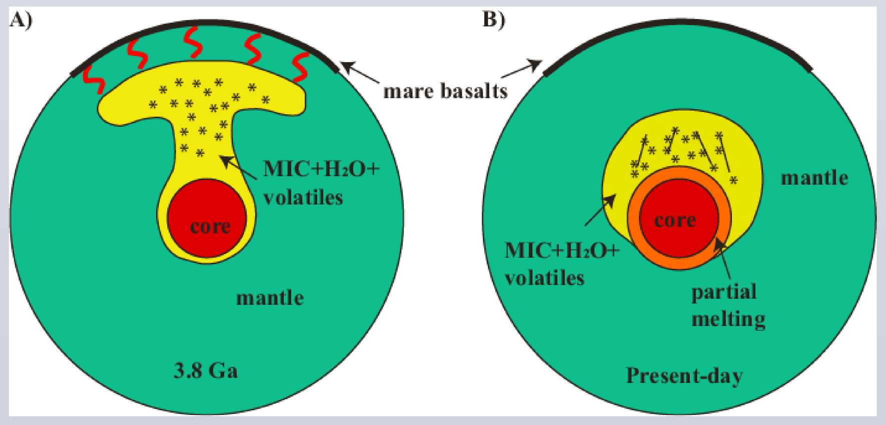
- The farside topography is several kilometers higher than the nearside, suggesting the farside has a thicker crust. [1]
- The mare basalts, formed by the most intense volcanism in lunar geological history, erupted predominantly on the nearside from ~3.9 Ga to ~3 Ga. [2]
- Deep moonquakes (DMQs), detected by Apollo seismic stations, are located mostly on the nearside at depths of ~800 km. [3]



- The hemispherical asymmetry of the crustal thickness and mare basalts distribution suggests long-wavelength lateral variability in lunar thermochemical structure at early stage[4,5,6], while the DMQ distribution reflects the present-day state of the lunar interior.

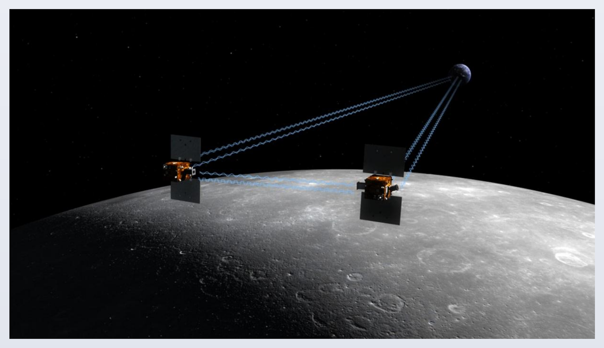
We hypothesize [7]:

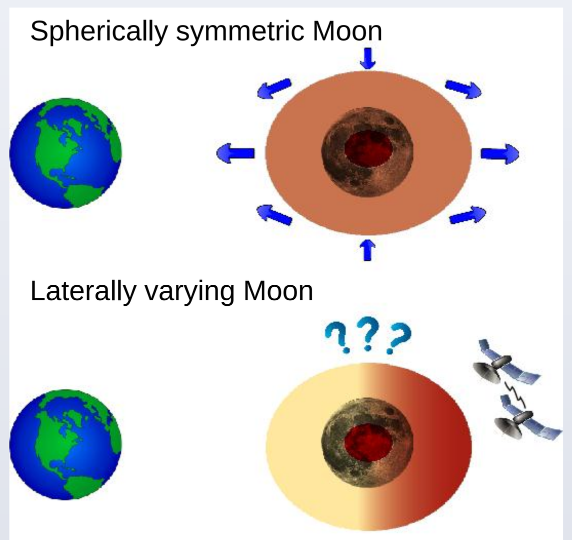
- The thermochemical evolution of the mixture of ilmenite cumulates and olivine-orthopyroxene (MIC) is the connection between the mare basalt emplacement ~3.8Ga and the DMQ in the present-day lunar mantle.
- The present-day lunar mantle still has the long-wavelength structure (e.g. the spherical harmonic degree 1 and order 1).



OBJECTIVES

The recently completed GRAIL mission provides lunar gravity field data to unprecedented precision (<1% at long wavelengths) [8]. We propose to use GRAIL observations to constrain the Moon's internal structure, by solving for tidal variations that would result from lateral variations in elastic moduli [9].





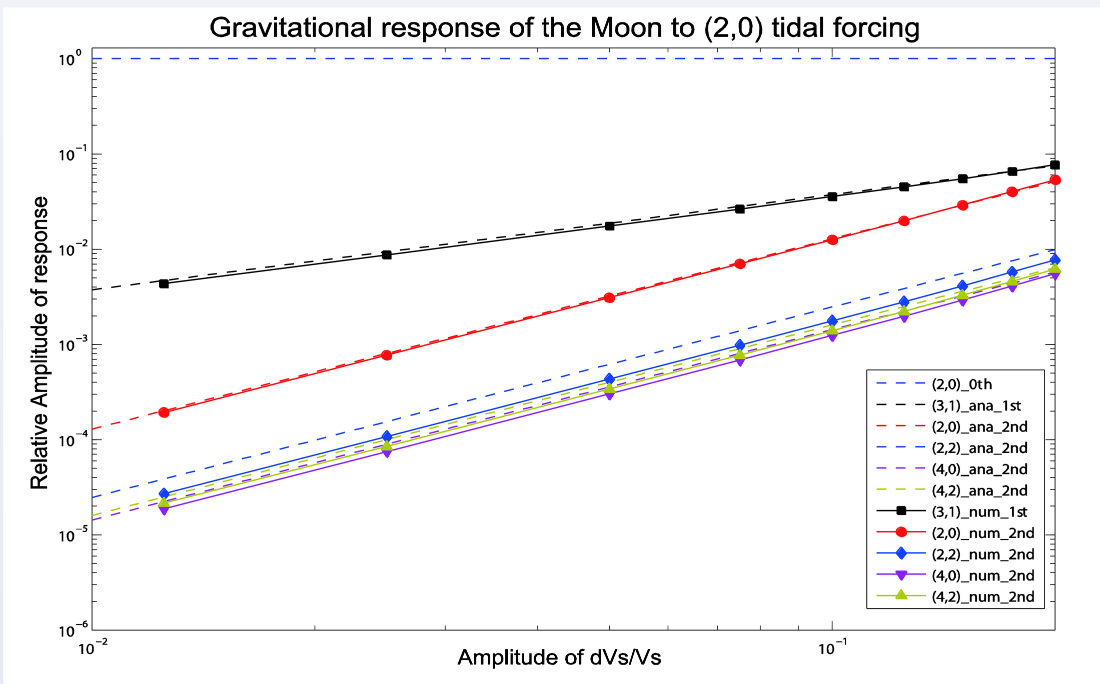
- When a degree-2 tidal force is applied to a spherically symmetric Moon, the tidal response must have the same spatial pattern. However, if lateral variations in elastic moduli (i.e. shear wave speed) of the lunar mantle exist, the tidal response will occur not only at degree-2 but also at other spherical harmonic degrees (i.e., mode coupling), with the amplitude of response depending on the wavelengths and amplitudes of the structural variations.
- In our former studies, we numerically determined the tidal response of a Moon with degree-1 laterally varying mantle structure (i.e. shear modules) using a finite element model [9].
- To better understand the physics of mode coupling between the tidal force and the laterally varying interior structures, and interpret our numerical results from a theoretical point of view, we here develop an analytic method based on perturbation theory [10] to complement the numerical approach.

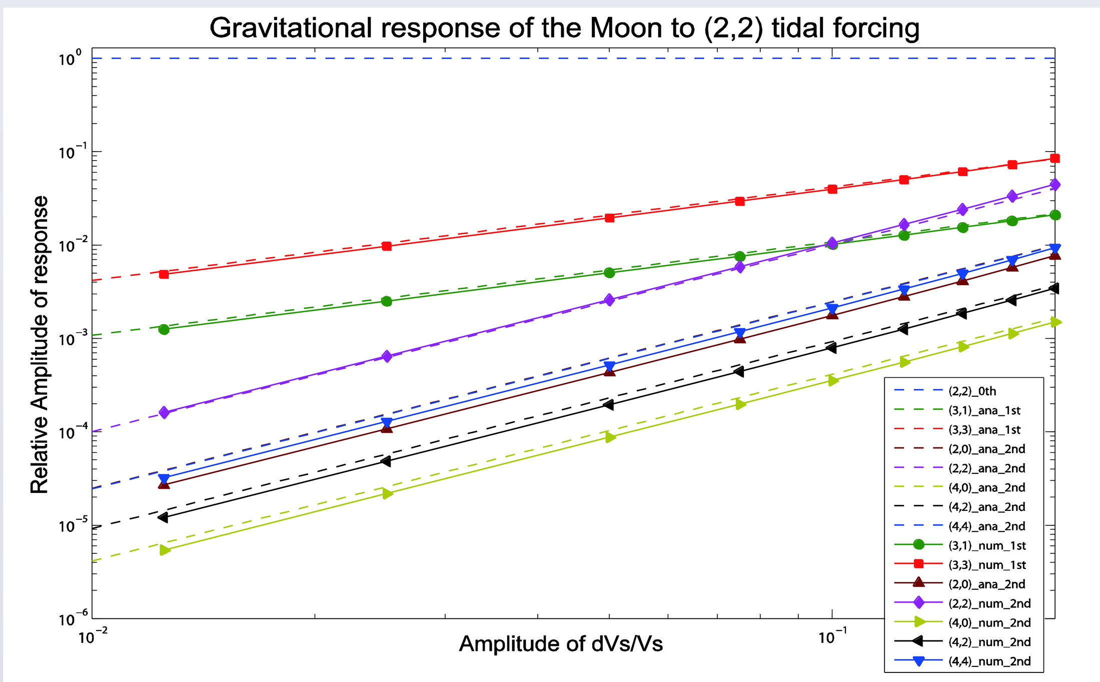
METHODS

- The lateral variation in the lunar mantle is assumed to be in spherical harmonic degree 1 and order 1 (l=1, m=1) and is introduced in mantle shear modulus μ in the form of dV_s/V_s, where V_s is the shear wave speed. The amplitude of dV_s/V_s is small and variable.
- The numerical calculation is done by the finite element code CitcomSVE that was originally developed to solve the post-glacial rebound problem for the Earth with a 3-D viscoelastic incompressible mantle [12, 13], and later modified to include compressibility [14].
- Analytically, we treat the small lateral variation as a structural perturbation to the 1-D background and rewrite the governing equations with such perturbation added.
- The perturbation theory works in the way that the equations can be split into similar forms in terms of different orders of perturbation. The major difference between them is the driving force, which determines at what spherical harmonic mode the tidal response would happen.
- For every order of perturbation, the equations are reformulated in terms of vector spherical harmonics expansion and are solved individually [15].

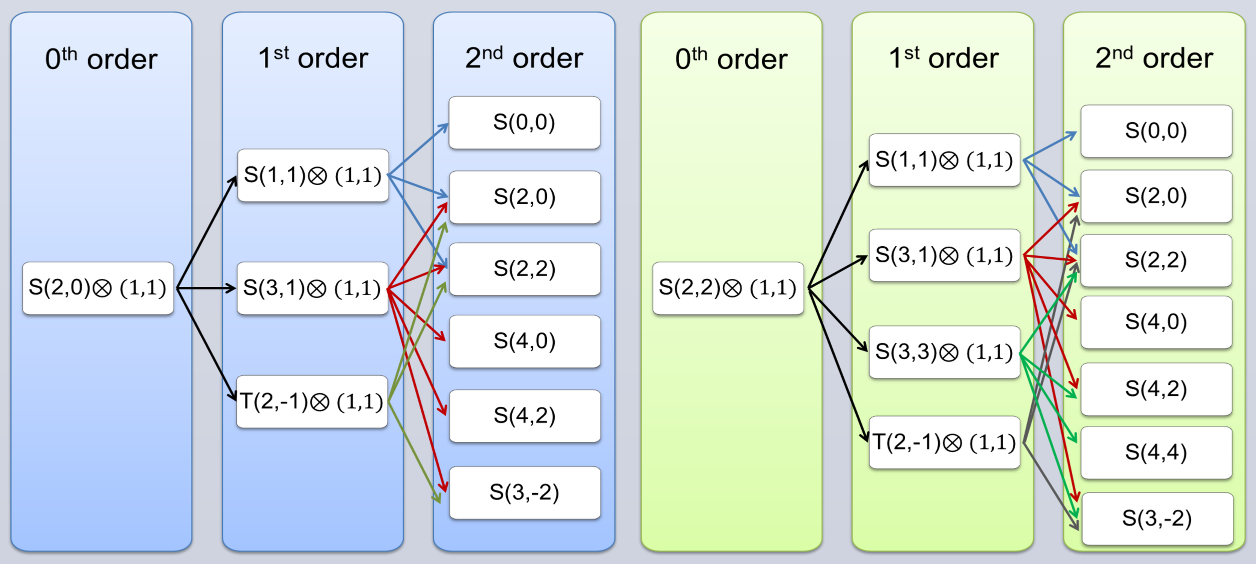
RESULTS

The comparison between the numerical and analytic results of the tidal response of the Moon to (l=2, m=0) and (l=2, m=2) tidal forcing, respectively.





- ❖ The marked solid lines and dotted lines are amplitudes of response predicted by numerical and analytic methods, respectively, with respect to the amplitude of dV_s/V_s, ranging from 1.25% to 20%.
- ❖ The diagrams below shows the mode coupling sequences predicted by the perturbation theory up to the second order, for the (l=2, m=0) and (l=2, m=2) tidal forcing, respectively. (S: spheroidal mode, T: toroidal mode)
- ❖ By our analytic prediction, the 1st and 2nd response increases linearly and quadratically, respectively, with respect to the increase of dV_s/V_s.



EQUATIONS

General equations:

- Tidal potential due to eccentric orbit of the Moon ($\cos(nt)$ term) [16]:

$$V_{td}(r,\theta,\phi,t) = \frac{3\varepsilon G m R_s^2}{4a^3}\left(\frac{r}{R_s}\right)^2 [\underbrace{\boxed{(1-3\cos^2\theta)}}_{Y_{20}} + \underbrace{\boxed{3\sin^2\theta\cos(2\phi)}}_{Y_{22}}]\cos(nt)$$

- Equation of motion [17]:

$$\nabla\cdot\overleftrightarrow{\tau} - \rho_0\nabla\phi - \rho_1 g_0\hat{r} - \nabla(\rho_0 g_0 u_r) + \vec{f}_{td} = 0$$

- Poisson's equation [17]:

$$\nabla^2\phi = 4\pi G\rho_1, \quad \textit{where } \rho_1 = -\nabla\cdot(\rho_0\vec{u})$$

- Constitutive relation for elastic medium [17]:

$$\overleftrightarrow{\tau} = \lambda(\nabla\cdot\vec{u})\overleftrightarrow{I} + \mu(\nabla\vec{u} + (\nabla\vec{u})^T)$$

When degree-1 lateral variation in shear modulus is introduced:

- Degree-1 lateral variation in shear modulus:

$$\mu(r) \rightarrow \mu(r,\theta,\phi) = \mu(r)(1 + \delta\cdot Y_{11}(\theta,\phi))$$

- 0th order equations:

$$\begin{cases}\nabla\cdot[\lambda(\nabla\cdot\vec{u}_0)\overleftrightarrow{I} + \mu(\nabla\vec{u}_0 + (\nabla\vec{u}_0)^T)] - \rho_0\nabla\phi_0 + \nabla\cdot(\rho_0\vec{u}_0)g_0\hat{r} - \nabla(\rho_0 g_0 u_{0r}) + \boxed{\vec{f}_{td}} = 0 \\ \nabla^2\phi_0 = -4\pi G\nabla\cdot(\rho_0\vec{u}_0)\end{cases}$$

tidal force

- 1st order equations:

$$\begin{cases}\nabla\cdot\left[\lambda(\nabla\cdot\vec{u}_1)\overleftrightarrow{I} + \mu(\nabla\vec{u}_1 + (\nabla\vec{u}_1)^T)\right] - \rho_0\nabla\phi_1 + \nabla\cdot(\rho_0\vec{u}_1)g_0\hat{r} - \nabla(\rho_0 g_0 u_{1r}) + \boxed{\vec{f}_{1st}} = 0 \\ \nabla^2\phi_1 = -4\pi G\nabla\cdot(\rho_0\vec{u}_1)\end{cases}$$

$$\boxed{\vec{f}_{1st} = \nabla\cdot[\mu\delta Y_{lm}(\nabla\vec{u}_0 + (\nabla\vec{u}_0)^T)]}$$

1st order force from mode coupling

- 2nd order equations:

$$\begin{cases}\nabla\cdot\left[\lambda(\nabla\cdot\vec{u}_2)\overleftrightarrow{I} + \mu(\nabla\vec{u}_2 + (\nabla\vec{u}_2)^T)\right] - \rho_0\nabla\phi_2 + \nabla\cdot(\rho_0\vec{u}_2)g_0\hat{r} - \nabla(\rho_0 g_0 u_{2r}) + \boxed{\vec{f}_{2nd}} = 0 \\ \nabla^2\phi_2 = -4\pi G\nabla\cdot(\rho_0\vec{u}_2)\end{cases}$$

$$\boxed{\vec{f}_{2nd} = \nabla\cdot[\mu\delta Y_{lm}(\nabla\vec{u}_1 + (\nabla\vec{u}_1)^T)]}$$

2nd order force from mode coupling

CONCLUSIONS & FUTURE WORK

- The degree-2 tidal force on the elastic and compressible lunar mantle with degree-1 lateral variation in shear modulus gives rise to non-degree-2 tidal response. It also affects tidal responses at degree 2. These results have important implications for using GRAIL observations to infer lunar interior structure including the core.
- Our analytic work based on perturbation theory has successfully predicted the specific modes arising from the tidal response of the Moon and interpreted the behavior of the amplitude of the response for each mode.
- Our analytic results agree reasonably well with our numerical results, especially for tidal response with relatively large signal, including 1st order degree-3 and 2nd order "self-coupling" responses.
- In our analytic work, we did not consider the special case of degree-1 tidal response from the first-order perturbation predictions. Since the degree-1 signal from the CitcomSVE is about two orders of magnitude smaller than degree-3 signal, we think it would have negligible influence on the second-order response. However, we will include the degree-1 analytic solution in order to fully resolve the problem.
- Our analytic framework is rather generalized and in principle can be applied to any laterally-varying structure. We plan to expand the work to consider different laterally varying lunar structure and predict the corresponding tidal responses.

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