Using Elastic Torque to Predict Libration on Icy Satellites

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Why we are interested:

- ✓ Motions of bodies can reveal internal structure
- ✓ Attractive Targets for Space Missions
- ✓ Origin of observed surface deformation are yet unexplained

What we are doing:

- ✓ Using **Elastic Restoring** to examine the motions of bodies with a surface shell decoupled from the interior by a fluid layer
- ✓ Analyzing the effects of viscous dissipation in the shells

What we are learning:

- ✓ Elastic Libration predicts a range of amplitudes
- ✓ Viscous layers reduce these motions
- ✓ Might build significant stresses

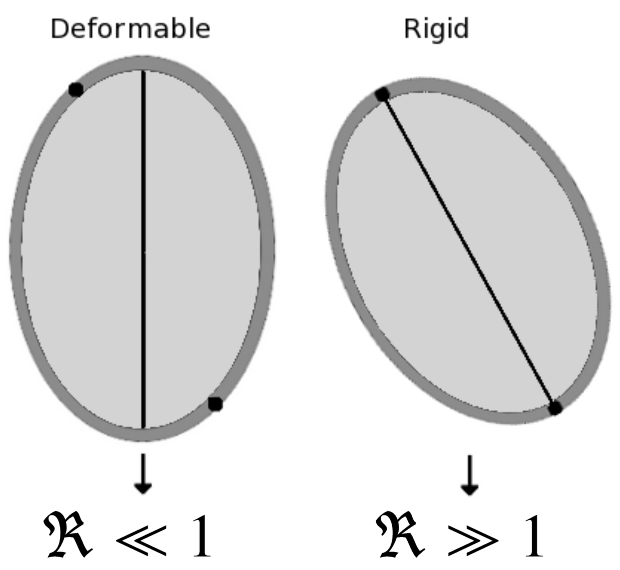
Elastic Torque and Response:

Two extremes can describe an ice shell's response to a rotation from equilibrium.

A rigid body will store gravitational energy

An elastic body will store gravitational energy. The ratio of these energies, which we use to determine the regime to study.

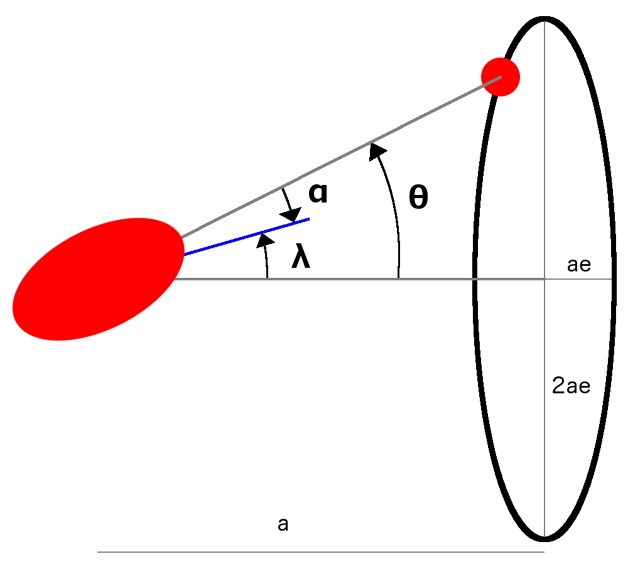
$$\Re = \frac{E_{elas}}{E_{grav}}$$



For both Europa and Titan, this value has been shown to be much less than 1 [1]. And so we consider such satellites as deformable

For a reorientation of an elastic ice shell by an angle α, the **restoring torque** is proportional to that angle [1]. This is confirmed using [2],[3].

$$\frac{\partial E(\alpha)}{\partial \alpha} = \tau(\alpha) = -k\alpha$$



The geometry of a synchronous moon in the guiding center reference frame is such that

$$\alpha = \lambda - \theta$$
$$\alpha = \lambda - 2e\sin(nt)$$

So we can now write

$$\tau(\lambda) = C\ddot{\lambda} = -k(\lambda - 2e\sin(nt))$$

This is the differential equation of a forced oscillator. The solution takes the form of classic **libration**.

$$\lambda(t) = \frac{2e\omega_0^2}{\omega_0^2 - n^2}\sin(nt)$$

With

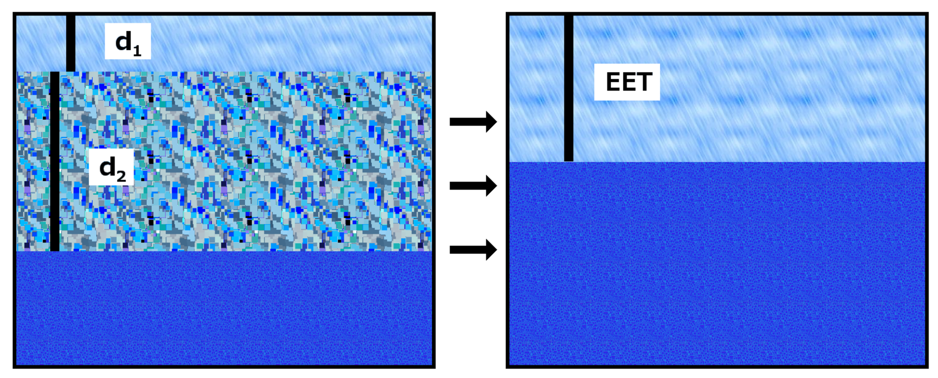
$$\omega_0^2 = \frac{k}{C}$$

Viscoelastic Layering:

With a completely elastic shell, we find that k is proportional to the shell thickness.

As we substitute viscous layers for elastic layers, we expect elastic energy to be reduced through viscous dissipation.

If we define a quantity, **"Effective Elastic Thickness" (EET)**, we can summarize a multilayer viscoelastic model by a single elastic layer.



We quantify EET discretely by

$$EET = d_1 e^{-\Delta_1} + d_2 e^{-\Delta_2}$$

And continuously with

$$EET = \int_0^d e^{-\Delta(x)} dx$$

We use the value Δ to describe a layer in the viscoelastic spectrum.

For $\Delta \ll 1$, the layer is elastic
For $\Delta \gg 1$, the layer is fluid

$$\Delta = \frac{\mu}{\eta\omega}$$

Notice that the expression for EET recovers the full shell thickness when completely elastic and decreases as a layer becomes more fluid.

With this EET, we can then use the elastic energy calculation of Goldreich and Mitchell (2010)[1] for a thin elastic shell.

$$k = \frac{32\pi}{5}\left(\frac{1+\nu}{5+\nu}\right)(1-k_{love})^2\left(\frac{nR}{GM}\right)^2\mu\left(R^3 - (R-EET)^3\right)$$

We write the moment of inertia, C, as

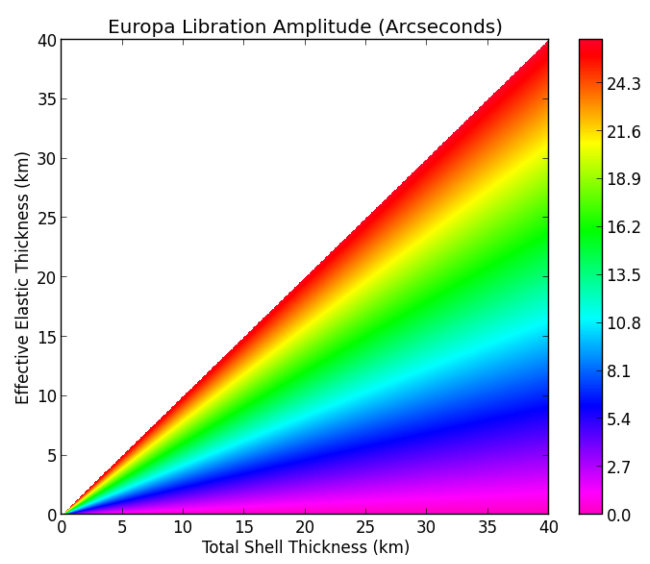
$$C = \frac{8}{15}\pi\rho\left(R^5 - (R-h)^5\right)$$

It follows that

$$\omega_0^2 = \frac{k}{C} \propto \frac{EET}{h}$$

Elastic Libration is a competition between elastic energy storage and the energy required to move the whole shell.

For Europa:



- Elastic torque predicts a range of amplitudes
- Amplitude displays the same trend as ω_0^2
- This plot displays magnitude. On Europa, elastic libration is 180° out of phase
- Interesting resonant behaviors possible

Implications:

With this model, measurements of rotation rate variation can **constrain rheological profiles** for ice shells

Dissipation of energy by viscous layers can tell us about the quantity of energy which goes into **tidal heating** on bodies.

Other Bodies:

- Ganymede
- Enceladus
- **Mercury**
- Titan

Layer Coupling:

How will individual layers couple to each other [4]?

- Gravity
- Pressure

Future Work:

Energy Loss:

How can we quantify viscous dissipation?

$$Q = 2\pi\frac{E}{\Delta E}$$

Can we compare to various rheology models?

- Maxwell
- Kelvin-Voight

How does this degree of tidal heating compare with that observed in the Galilean Satellites?

Surface Stress:

If the shell rotates at a different rate then the rest, reorientation stresses should act through the shell.

- Magnitudes
- Patterns

How does this process relate to the often invoked **"Non-Synchronous Rotation"**?

References:

[1] P. M. Goldreich and J. L. Mitchell. Elastic ice shells of synchronous moons: Implications for cracks on Europa and non-synchronous rotation of Titan. *Icarus*, 209:631–638, October 2010.

[2] Z. A. Selvans. *Time, tides and tectonics on icy satellites*. PhD thesis, University of Colorado at Boulder, 2009.

[3] J. Wahr, Z. A. Selvans, M. E. Mullen, A. C. Barr, G. C. Collins, M. M. Selvans, and R. T. Pappalardo. Modeling stresses on satellites due to nonsynchronous rotation and orbital eccentricity using gravitational potential theory. *Icarus*, 200:188–206, March 2009.

[4] T. van Hoolst, N. Rambaux, Ö. Karatekin, V. Dehant, and A. Rivoldini. The librations, shape, and icy shell of Europa. *Icarus*, 195:386–399, May 2008.