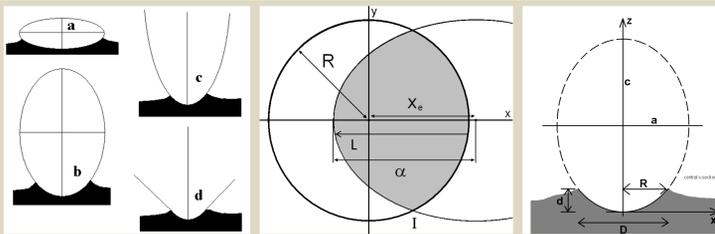


Introduction

Chappelow [1,2,3] developed an “enhanced” shadow measurement technique that allows determination of simple crater shape even if the shadow does not pass through the deepest part of the crater. I test the methodology using a well-imaged and well-constrained lunar crater, and then I show initial observations from four outer planet satellites.

Enhanced Shadow Measurement Method

In a series of papers, Chappelow [1,2,3] developed an “enhanced” shadow measurement technique that allows determination of crater shape (parabolic, hemispheric, etc.) and rim-floor depth d for any crater whose shape is a conic section of revolution; simple craters on the planets are generally well described by this shape. A key advantage of the method is that the shadow front does not need to pass through the crater center (i.e., the deepest point in the crater) in order to accurately determine depth. The methodology involves fitting an ellipse to the shadow front, and the shape and position of the ellipse uniquely determine the shape of the conic section and the crater depth.



If the crater interior can be described by a standard conic section (left), then with the sun in the positive x-direction, α and R are the semi-axes of the ellipse and x_c is the displacement from the crater center. The shadow length L and the rim intersection point I can be measured to solve for x_c and α . Looking at the crater in cross-section (right), parameters a and c define the shape of the conic section (Figures adapted from [2,3]).

The general equation for the conic section, assuming z defines the vertical axis from crater center, is

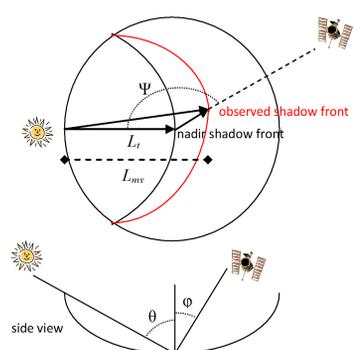
$$\frac{x^2 + y^2}{a^2} + \frac{(z - c)^2}{c^2} = 1$$

where a and c are the semi-axes of the conic section describing the crater shape. On the crater rim $x^2 + y^2 = R^2$ and $z = d$. Measurement of the shadow length L and its intersection point with rim (I_x, I_y) allows one to solve for x_c and α , and in turn a and c . The sign and values of a and c , which can be imaginary, determine the shape of the conic section. The method originally was restricted to nadir-pointing images, but subsequently a method was developed to account for the parallax of the shadow tip with non-zero emission angles.

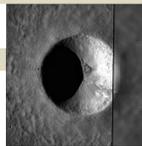
For imaging that is off-nadir, after the image is map projected the shadow tip needs to be corrected for parallax relative to the crater rim. If θ is the solar incidence angle, ϕ is emission angle, ψ is the angle between their azimuths, and L_{mx} is the component of the measured shadow length in the direction of the sun, then

$$L_t = L_{mx} \frac{\tan \theta}{(\tan \theta + \tan \phi \cos \psi)}$$

where L_t is the “true” shadow length that would be measured from a nadir looking perspective.



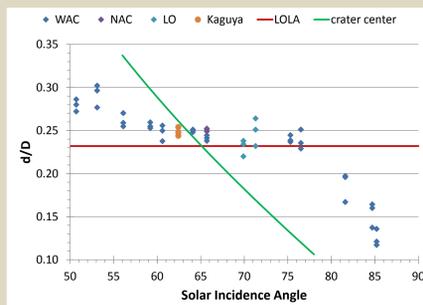
LRO-NAC image of mare crater ($D = 3.8$ km, 1.8 S, 351.8 E) superposed on WAC image.



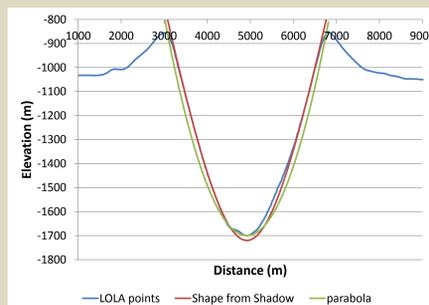
Testing with a lunar mare crater

In analyzing craters on Mercury [4] with the method, we noticed a correlation of depth with incidence angle. I decided to evaluate this and the overall consistency and accuracy of the method by examining a $D = 3.8$ km lunar mare crater (1.8 S, 351.8 E) for which there were numerous images taken by multiple spacecraft with a variety of incidence and emission angles and directions. For each image I made three measurements. Our observations lead to the following conclusions.

- Measurements are valid for shadows that extend from the rim to $\sim 0.3 - 0.75 D$. Within this range a $\pm 5\%$ trend with θ may exist.
- With the parallax correction, there is no reduction in accuracy for off-nadir images (highest emission angle tested was 35° for LO image)
- Within recommended range of θ , measurements were within $\pm 10\%$ of median; I consider this the error associated with a single measurement.



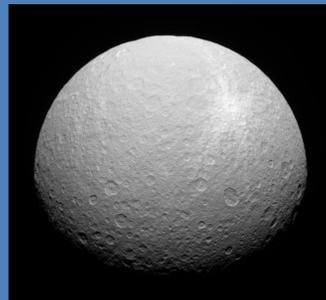
Three enhanced shadow measurements were taken from images spanning a variety of resolutions, spacecraft, incidence angles, and emission angles. Horizontal line is depth from LOLA profile that just misses crater center. Green line is d/D for shadow that crosses crater center at given incidence angle.



LOLA profile that just misses crater center, along with shape determined from shadow measurement method, and the shape of a parabola with the same depth as the LOLA profile. The crater is slightly more cone-shaped than a parabola.

Results for Icy Satellites

Using enhanced shadow measurements, I have begun a survey of craters on outer planet satellites. These are objects for which no altimetry exists and stereo-derived topography is generally limited in resolution. I began the survey by looking at four bodies: Rhea, a largely inactive moon of Saturn that should provide a baseline for “typical” icy satellite depths; the Uranian moon Miranda, whose initial d/D values [5] now seem at odds with Galileo-era measurements [6] of other moons; and Hyperion and Phoebe, two small, irregularly shaped moons of Saturn, the latter thought to be a captured body [7].



Rhea ($D = 1528$ km)

For Rhea, I made measurements from an initial set of three Cassini ISS images, one at a resolution of 40 m/pixel and two at 260 m/pixel. The data span $300 \text{ m} < D < 12 \text{ km}$ and show no obvious trend of d/D with D . The mean value of d/D is 0.197 ± 0.032 . This is consistent with stereo-derived depths for Rhea in [8] and simple craters on the Galilean satellites in [6] (and the terrestrial planets), but higher than those for Rhea in [9]. I also do not see the drop in d/D for $D < 1 \text{ km}$ that is described in [9]. Along with d/D , the inferred shapes vary, but an average crater on Rhea is parabolic.

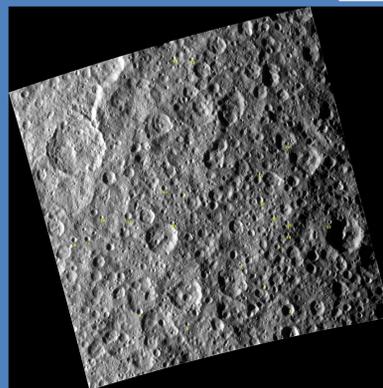
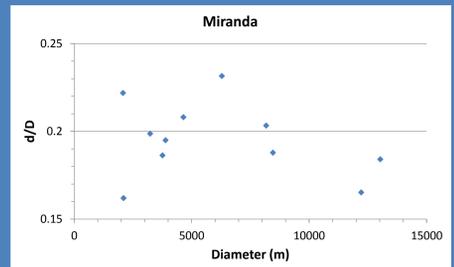
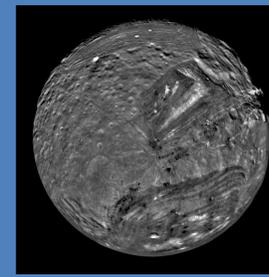
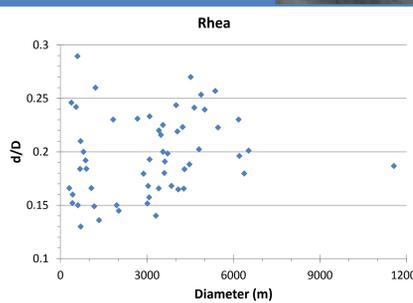


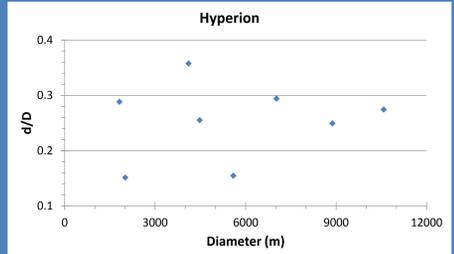
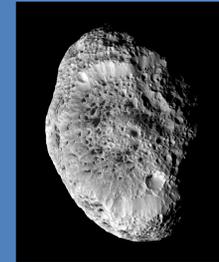
Image of portion of Rhea showing craters for which shadow measurements were made. Image is 335 km across and 263 m/pixel at full resolution.

Shadow fit to $D = 2$ km crater on Rhea. That the fit ellipse is nearly circular indicates that the crater is parabolic.



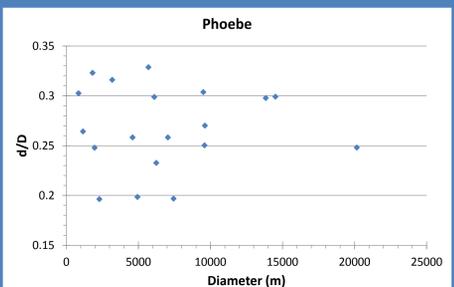
Miranda ($D = 470$ km)

Two Voyager frames with resolutions of 240 and 270 m/pixel were used for Miranda. For $2 \text{ km} < D < 13 \text{ km}$, there is no clear dependence of d/D with D . The mean value of d/D is 0.195 ± 0.016 . These d/D values are consistent with other “typical” icy satellites, but higher than the initial post-Voyager Miranda measurements of [4], which were primarily based on photogrammetry. The mean shape information suggests craters slightly more conical than parabolic (similar to the lunar example), but the spread in the shapes is high enough that a mean parabolic shape cannot be ruled out.



Hyperion (360 x 266 x 205 km)

Incidence angles for many of the shadows on Hyperion cannot be reliably estimated because the moon's highly irregular shape makes local slopes deviate from the spheroidal model assumed in the ISIS software. I found one image where the spacecraft facing surface was relatively well-behaved with good shadows, and I was able to determine a handful of d/D values (Figure 6). $1.8 < D < 10.6 \text{ km}$, no dependence of d/D on D is observed, and the mean d/D is 0.253 ± 0.051 . Although consistently deep, the crater shapes are parabolic.

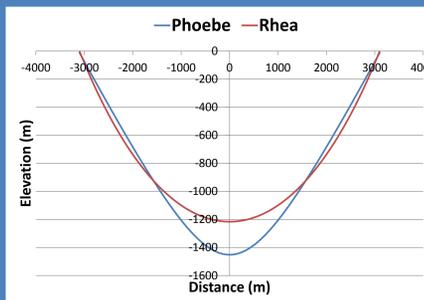


Phoebe (219 x 217 x 204 km)

Two Cassini images with resolutions of 89 and 194 m/pixel were used to examine Phoebe's craters. For $850 \text{ m} < D < 20 \text{ km}$, there does not appear to be a dependence of d/D with D . The mean value of d/D is 0.268 ± 0.035 , consistent with stereo-derived profiles in [9]. All of the craters measured that are above a few km in diameter are notably more hyperbolic (cone-shaped) than parabolic in their interior.

Crater Shape Comparison

Comparison of shapes and depths for $D = 6.2$ km craters on Phoebe (left) and Rhea (right). The crater on Phoebe has slightly steeper walls and is more cone shaped than the nearly parabolic crater on Rhea.



Discussion

The enhanced shadow measurement method produces reliable results and should prove particularly valuable for studying icy moons and asteroids. Rhea and Miranda generally have “typical” simple craters with d/D similar to the Galilean satellites and the terrestrial planets. Phoebe and Hyperion both seem to have unusually deep craters, although probably from different causes. Craters may be strength-controlled on Phoebe, while Hyperion is an exceptionally porous target.