A PROBABILISTIC MODEL TO EXPLORE DEPTH-DIAMETER DEPENDENCIES FOR LUNAR CRATERS P. Mahanti[1], M.S. Robinson[1], R. Stelling[1], S.J. Lawrence[1], J. Stopar[1] [1]Lunar Reconnaissance Orbiter Camera, School of Earth and Space Exploration, Arizona State University, Tempe,AZ,USA; pmahanti@asu.edu;

Introduction: Depth-diameter relationships of lunar craters were studied as early as 1963, and deterministic models (with uncertainty margins) such as the power law were used historically in the works of Baldwin, Pike, Elachi and others [1, 2, 3, 4]. These models were useful in the context of the Apollo era data, but as this work shows, alternate mathematical models better represent reality especially in light of new high resolution ranging and stereo observations from robotic missions. Here we employ a probability density model to fit the statistical distribution of the new depth-diameter data instead of classic regression methods. The resulting statistical distribution (probability density function) jointly describes the observed trends and dependencies of depth and diameter without enforcing any mathematical expression. We show that the model accurately portrays data density and that the model can resolve conditional queries. Cause and effect parameters can be easily added to the basic model, ultimately leading to a robust Bayesian network (future work) that can be effectively utilized for a unified study of the lunar cratering process.

Background: Relationships between depth (R) and diameter (D) were described by Baldwin [1] as a 3rd order polynomial in log-domain. Pike [5],suggested a power law was a better description of the relationship between these quantities. Since then, the Apollo data has been used to fit both power law [6] and linear relations [3]. While isometric growth was suggested [5] as a reasoning for a power-law relationship, this and other relationships are not causal and do not represent a probabilistic model. So, given a set of data points we can either explore multiple possible deterministic relationships with yet undetermined confidence zones (since topographic data is still being collected, and of better quality) or a single probabilistic model that does not hypothesize a relation between physical quantities.

Data and methods: Historical data used in this work comes from Apollo 15-17 missions. The values of the depth and rim-to-rim diameter and the relationships between rim-height and rim-to-rim diameter are used to obtain apparent depth and apparent diameters of 118 craters (1km<D<10km)[7]. New data for 540 craters (D<150m) was collected in this work from Lunar Reconnaissance Orbiter Narrow Angle Camera (LRO NAC)-Digital Elevation Models (DEMs)[8]. Statistical analysis, probabilistic modeling and robust fitting was performed using in-house algorithms and the Matlab statistical toolbox (Mathworks Inc.). Linear($R = mD + c$) and power law($R = aD^b$) relations are fit to Apollo data and LRO data individually and together (Fig.1 and Table 1). For these data, linear and the power law fits have the similar goodness-of-fit and indicates that none these models are unique, and an even better model is always possible. Moreover, we find from earlier work that the power law relationship had different coefficients across various diameter ranges and similar behavior is expected for the linear model - so there are different deterministic models with multiple variations of the same model.

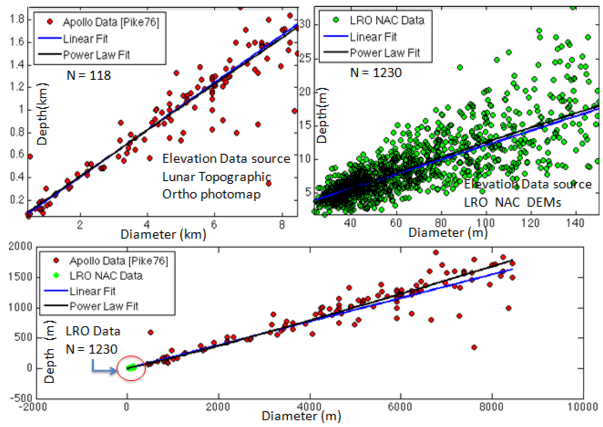

Figure 1: Power Law and Linear fitting of crater depth vs diameter for Apollo and LRO elevation data

Table 1: Fitting performance of deterministic relations

	Power Law fit	Linear fit
Apollo data	a = 0.205,b=1.001 R-square: 0.995 RMSE: 0.037	m = 0.211,c = -0.019 R-square: 0.952 RMSE: 0.108
LRO data	a = 0.219,b=0.879 R-square: 0.983 RMSE: 0.592	m = 0.108,c = 1.379 R-square: 0.757 RMSE: 2.253
Composite	a = 0.095,b=1.089 R-square: 0.999 RMSE: 3.287	m = 0.194,c = -3.392 R-square: 0.999 RMSE: 4.579

A new probabilistic model: As an alternate method of understanding crater formation and associated morphology we propose modeling a probabilistic relationship. Reasons for this are : (a) It is not deterministic, so it does not coerce us into believing some form of relationship which may not be true as more data becomes available over time ;(b) A probabilistic model generated from data can be used to infer deterministic

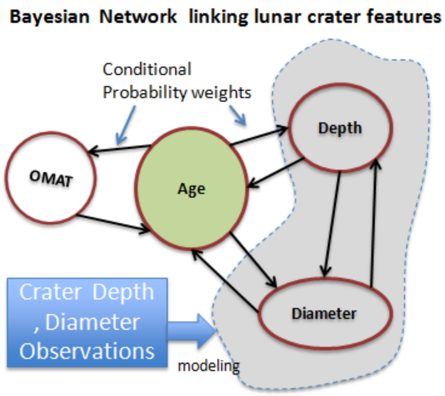

Figure 2: A Bayesian network example

relations, and then improving the model based on these relations; (c) The model allows incorporation of additional parameters when required which allows us to probabilistically relate cause and effects and also determine which causes (and effects) are more likely than others. This ultimately leads to a Bayesian network which operates on the basis of conditional probability. It may be noted that the current modeling of depth-diameter is not a cause and effect model, but causes (e.g. impact velocity from simulation studies) and effects can be added to this model.

The schematic of a possible Bayesian network is shown in Fig.2. The arrows indicate the direction of dependency. For each of these arrows we can have a weight which are conditional probability values obtained from the probabilistic model. A lower weight indicates less dependence. For example, a young crater may strongly indicate a high optical maturity (OMAT) value but the converse may not be true. Again the age may strongly indicate degradation and change in diameter, but it may not strongly indicate changes in depth. Also, OMAT values and diameter values are not expected to have any dependency, and not linked in this network. Note that all these parameters cannot be easily linked by a single deterministic equation. However the probabilistic model allows this, query the model and get answers based on and guided by data density. From the modeling perspective this is a more unifying approach than fitting data to equations.

An example of probabilistic modeling for Apollo data and LRO data is shown in Fig.3. The importance of data density is immediately evident from the figure. A linearly related trend can be observed but this is imposed and controlled by the probability contours. The zones colored red, imply stronger confidence of dependence and comparing the two plots we can see that we can trust the LRO data for small crater relations and Apollo data for large craters, while in between, data density is sparse and there is low probability of any dependence (even though it might actually exist).

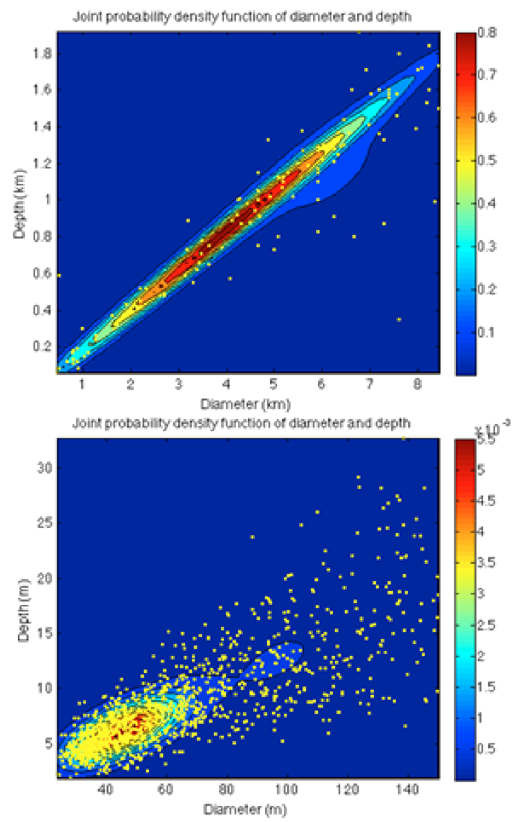

Figure 3: Probabilistic models for Apollo and LRO data

Conclusion: Probabilistic modeling of crater depth-diameter relationships is explored in context of the new data available from the LRO DEMs. The proposed model is strongly dependent on data density and is not based on any single equation. Once developed such a model can accommodate additional factors through conditional probability weights in a Bayesian network architecture. We hope to gain from this model additional insight into cratering mechanisms and linkages between crater morphology, spectral properties and crater degradation.

References: [1] R. Baldwin (1963) *[Chicago] University of Chicago Press [1963]* 1. [2] R. Baldwin (1965) *New York, McGraw-Hill [1965]* 1. [3] C. Elachi, et al. (1976) *Earth, Moon, and Planets* 15(1):119. [4] R. Pike (1977) in *Lunar and Planetary Science Conference Proceedings* vol. 8 3427–3436. [5] R. Pike (1967) *Journal of Geophysical Research* 72(8):2099. [6] R. Pike (1977) in *Impact and Explosion Cratering: Planetary and Terrestrial Implications* vol. 1 489–509. [7] R. Pike (1976) *Earth, Moon, and Planets* 15(3):463. [8] T. Tran, et al. (2010) in *Special joint symposium of ISPRS Technical Commission IV and AutoCarto* 15–19.