SENSITIVITY ANALYSIS FOR HAPKE'S RADIATIVE TRANSFER MODEL. D. Liu[1], L. Li[1], Y. Z. Zhang[2], [1]Department of Earth Sciences, 723 W. Michigan Street, Indiana University-Purdue University, Indianapolis, IN, 46202, email: liudawe@imail.iu.edu and ll3@iupui.edu, [2]Institute of Space and Earth Information Science, the Chinese University of Hong Kong, Hong Kong, email: yuanzhizhang@cuhk.edu.hk

Introduction: Remote determination of the mineral abundance on the lunar surface is of significant importance for studying lunar geology [1-2]. To date, Hapke's Radiative Transfer Model (RTM) is the most widely used method that allows for simulating the reflectance spectra of lunar soils and for inverting mineral abundance because of its capacity in accommodating the effects of space weathering and viewing geometry [3-4]. The variation of simulated reflectance by Hapke's RTM is driven by a variety factors, including particle size (PS), the abundances of submicroscopic iron (SMFe) and minerals. In addition, optical constants of minerals, viewing geometry, the form of phase function (P(g)) and back scattering function (B(g)) also affect the simulation via Hapke's RTM. However, quantitative analysis of relative importance of these factors to the performance of Hapke's RTM has never been conducted though such analysis could help to determine the factors that interfere with spectral estimation of lunar mineral abundance. In this study, the sensitivity of Hapke's RTM simulated reflectance to the variation of SMFe, PS and mineral abundances was quantitatively analyzed via Extended Fourier Amplitude Sensitivity Test (EFAST) with the aim of determining the relative significance of these factors in regulating the simulated reflectance.

Dataset: Spectral reflectance simulation via Hapke's RTM needs input factors such as, PS, SMFe, mineral abundance, view geometry, P(g), B(g), optical constants of minerals and Iron. Here, PS, SMFe and the abundances of typical lunar soil minerals were set as input variables for sensitivity analysis while other contributing factors were fixed to known values [5]. For PS, its variation ranged from 1 μm to 45 μm, and SMFe ranged from 0% to 1%. Four composing minerals were considered: plagioclase, orthopyroxene, clinopyroxene and olivine. The abundances of these minerals were assumed to vary within the range of 0%-100%. All these variables were assumed to be uniformly distributed. Each of six variables (one for PS, one for SMFe and four for mineral abundance) of a sample was randomly selected from the corresponding variation range of that variable and the sum of mineral fractions of each sample was set to one. In total, 390 samples with different PS, SMFe and mineral abundances were simulated via Hapke's RTM.

Extended Fourier Amplitude Sensitivity Test (EFAST): Sensitivity analysis (SA) is to apportion the variance in the output of a model to different sources of variation and to identify the factors that mostly contribute to output variability [6]. EFAST is a variance-based SA. EFAST calculates total sensitivity index (TSI), which not only measures main contribution of each input factor to the output variance, but also takes into account the effects of interactions among factors [7]. To illustrate SA clearly, reflectance spectra $\boldsymbol{Y_{rs}}$ are used as an example. Assuming that $\boldsymbol{Y_{rs}}$ is only controlled by particle size ($\boldsymbol{X_{ps}}$), and the abundances of SMFe ($\boldsymbol{X_{Fe}}$) and minerals ($\boldsymbol{X_{mi}}$). The variance of $\boldsymbol{Y_{rs}}$ can be decomposed via EFAST as following:

$$\mathrm{V}(Y_{rs}) = V(X_{ps}) + V(X_{Fe}) + V(X_{mi}) + V(X_{ps}, X_{Fe}) + V(X_{ps}, X_{mi}) + V(X_{Fe,} X_{mi}) + V(X_{ps}, X_{Fe}, X_{mi})$$

where $\mathrm{V}(Y_{rs})$ is the variance of Y_{rs}, $V(X_{ps})$ is the variance of X_{ps}, $V(X_{Fe})$ is the variance of X_{Fe}, and $V(X_{mi})$ is the variance of X_{mi}. $V(X_{ps}, X_{Fe})$, $V(X_{Fe,} X_{mi})$ and $V(X_{ps}, X_{mi})$ represent the variance of interaction between X_{ps} and X_{Fe}, $X_{Fe}$and X_{mi}, X_{ps} and X_{mi} respectively. $V(X_{ps}, X_{Fe}, X_{mi})$ represents the variance of interaction among X_{ps}, X_{Fe} and X_{mi}. The TSI of X_{ps} is defined as:

$$TSI_{X_{ps}} = \frac{V(X_{ps}) + V(X_{ps}, X_{mi}) + V(X_{ps}, X_{Fe}) + V(X_{ps}, X_{Fe}, X_{mi})}{\mathrm{V}(Y_{rs})},$$

which represents the contribution of PS to the reflectance variance.

EFAST was applied to 390 simulated reflectance spectra at each wavelength between 400 nm and 2500 nm (with 5 nm intervals) to determine the sensitivity of Hapke's RTM to the input PS, SMFe, and abundance of minerals [5].

Results and Discussion: Shown in Figure 1 are TSI values for all the input factors. TSI values for orthopyroxene and clinopyroxene were added together. It can be seen that the simulated reflectance is highly sensitive to PS and SMFe at all wavelengths. In contrast, the abundance of minerals merely accounts for a small portion of variation of output reflectance. The significant effect of PS at all wavelengths is expected because the variation of PS strongly affects the magnitude of reflectance, which contributes mostly to the variation of a spectrum. Compared to other spectral regions, PS shows the largest influence around

1000 nm and 2000 nm. One reason might be that these two regions are absorption wavelength of mafic minerals. Change in PS not only affects the magnitude of a spectrum but also the absorption depth [8]. SMFe also exhibits significant importance in regulating the reflectance due to its spectral effects of reddening, darkening and reducing spectral contrast [9]. However, the sensitivity of simulated reflectance to SMFe decreases apparently close to 1000 nm and 2000 nm, which can be explained by several reasons. First, these two spectral regions are strongly affected by the mineral absorption and abundance of minerals contributes more to these two regions, ultimately weakens the influence of SMFe. Second, *as* stated in [10], when high maturity of sample is present, the effect of SMFe on absorption depth will be saturated and variation of SMFe may no longer affect spectral absorption. Third, a negative correlation exists between PS and SMFe (as shown in Fig. 1a). This is consistent to what happens in lunar soil saturation: reducing PS and generating SMFe. When reflectance shows high sensitivity to PS at absorption wavelengths, the contribution from SMFe is relatively low.

The TSI curves for mineral abundances bear resemblance to the absorption spectrum of the corresponding mineral. The TSI curve for plagioclase is flattened across the whole spectral region because plagioclase exhibits low spectral contrast and the variation of plagioclase abundance reflects the same case via TSI. Although the sensitivity of simulated reflectance to plagioclase is not comparable to that for PS and SMFe, it is still much higher than that for the abundances of pyroxene and olivine due to its significant influence on the magnitude of reflectance. For pyroxene, the TSI values are relatively higher near 1000 nm and 2000 nm because of Fe^{2+} absorption of pyroxene at these two wavelengths. The variation of olivine abundance accounts for a very small portion of reflectance with only a flat peak value between 1050 nm and 1300 nm. The sensitivity analysis results suggest that simulated reflectance is more sensitive to the abundance variation of plagioclase and pyroxene than that of olivine. Thus, when Hapke's RTM is applied to quantify lunar soil minerals, higher accuracies could be obtained for plagioclase and pyroxene than olivine [11].

Conclusion: Application of EFAST to the reflectance spectra via Hapke's RTM shows that PS and SMFe are the two most important factors that drive the variance of lunar soil reflectance. The contribution of mineral abundances to reflectance is much less significant than that from PS and SMFe, and could be suppressed by PS and SMFe. This creates a challenge for deriving lunar mineral abundance from remotely sensed data. In order to estimate mineral abundance more accurately, continuum removal could be first applied to measured reflectance spectra to remove the effects of PS and SMFe. In addition, when Hapke's RTM is applied to derive the abundance of minerals in lunar soils, the estimation accuracy could be potentially improved by choosing specific spectral regions where minerals contribute more to the variation of reflectance, such as 1000 nm and 2000 nm for pyroxene, and 1050 nm to 1300 nm for olivine.

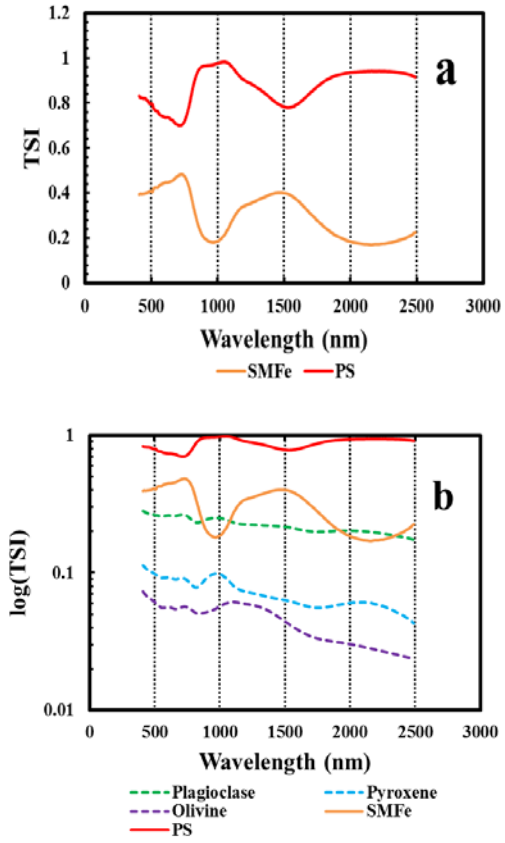

Figure 1 TSI values for Hapke's RTM (a) TSI for PS and SMFe (b) log scale of TSI for all input factors

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