A NEW METHOD FOR DETERMINATION OF CRATER SHAPES AND DIMENSIONS USING SHADOWS: PROOF OF CONCEPT. JE Chappelow[1], [1]Meteorifics Inc., POB 751603, Fairbanks, AK, 99775 (john.chappelow@saga-inc.com).

Introduction: In two previous abstracts [1,2] I described a new method for using shadows cast in simple craters to determine their shapes and dimensions using the fact that the shadow cast inside a conic-section shaped crater must itself be bounded by an ellipse (Fig.1). Equations derived from this observation were given, relating the shape of the shadow inside such a crater to its cross-sectional shape [1]. I also noted that what was still needed was the inverse of these equations; ie. the crater shape in terms of the shadow shape parameters (which can be measured). I have now finished the math for this method, implemented it in the form of a computer program, and tested it against a DEM of a well known impact crater. Here I report the results.

Shape-from-Shadows Equations: Inverting the equations given in [1] results in three equations that define the conic-section crater shape in terms of shadow-related quantities (R, α, and x_c) measurable in spacecraft imagery:

$$a^2 = \left(\frac{R^2 - \alpha^2 + x_c^2}{R^2 - \alpha^2}\right) R^2 \qquad (1)$$

$$c^2 = \left(\frac{R}{(\alpha - R)\tan\theta}\right)^2 \left(R^2 - \alpha^2 + x_c^2\right) \qquad (2)$$

and (left in terms of a and c):

$$d = c - \left(\frac{a^2 + c^2 \tan^2\theta}{2a^2 \tan\theta}\right) x_c \qquad (3)$$

where a and c are the vertical and radial semi-axes of the crater shape conic-section, R and d are the crater radius and depth, and θ is the solar incidence angle; α and x_c are properties of the shadow boundary ellipse as defined in Fig.1(a), and along with R, are measurable parameters. The quantities a, c and d from Eqs.(1)-(3) are sufficient to define the approximating conic-section, however depth d, crater diameter D, and the eccentricity of the crater shape conic-section, e, are more convenient and intuitively meaningful. The eccentricity can be obtained from:

$$e = \sqrt{1 - a^2/c^2} \qquad (4)$$

I have written a computer program to apply these equations. It asks the user to select three points on the crater rim, fixing the position and radius of the crater rim circle, and then two more on the shadowfront, which together with other constraints are sufficient to fix the shadowfront ellipse, and α and x_c. It then calculates all crater shape related quantities.

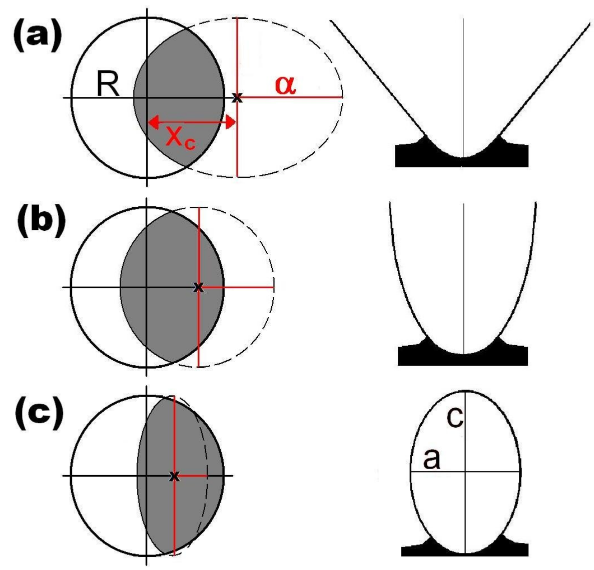

Fig.1: Shadow shapes (left) and corresponding crater shapes (right). On the right, the solid black circle is trhe crater rim, the dashed ellipse is the shadow boudary ellipse. The crater shapes to the right are not to scale, and are for illustration only. Illumination is from the right. Key:

Crater shape	**(a)** hyperbolic	**(b)** parabolic	**(c)** elliptical
e	$e > 1$	$e = 1$	$e < 1$
α	$\alpha > R$	$\alpha = R$	$\alpha < R$

A Reality Check - Linne Crater: A Lunar Reconnaissance Orbiter Wide Angle Camera image covering Linne (m162229369) was obtained from the Planetary Data System, along with a DEM sampled at 8 m/pix. I used the computer program to determine the approximating conic-section for the crater (D = 2210m, d = 541m, e = 1.62), then took 72 radial cross-sections of Linne from the DEM (center-rim, spaced 5°), computed their average, and then plotted all three.

Figs.2 demonstrate the very close agreement between the crater shape determined using shadow measurements and the actual crater profile. The only significant divergence between the two is the flat crater bottom, which cannot be approximated by any conic-section. However the difference here is minor (Fig.2a), and unless they are completely hidden in shadow, flat bottoms are easy to detect and treat separately. And the approximating conic also provides a good estimate of the bottom fill in the crater (in this case 20-30m).

Though Linne is often described as "parabolic" in shape, the eccentricity value (1.62) is that of a hyperbola, and not that of a parabola or a cone.

References: [1] Chappelow JE (2008), *39th LPSC*, Abst. #1441.[2] Chappelow JE (2008), AGU Fall Meeting 2008, Abst. #P31A-138.

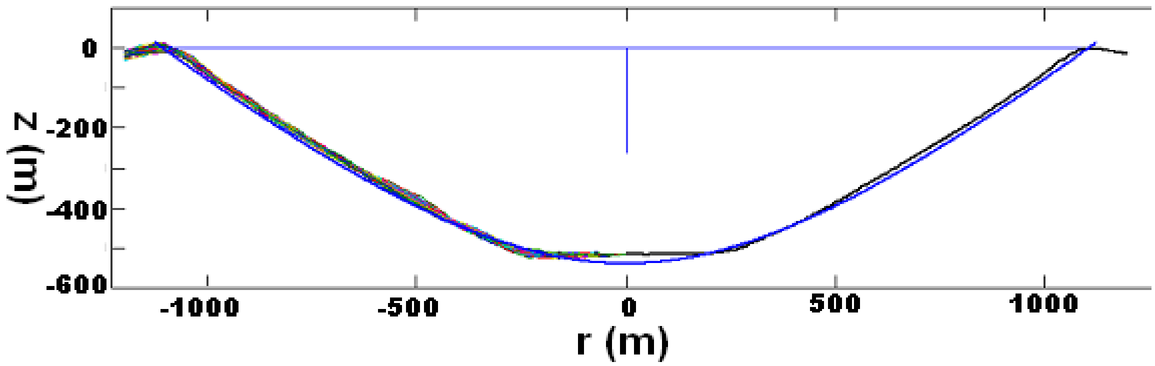

Fig. 2a: The approximating hyperbola (blue) plotted over the 72 radial profiles of Linne (various colors, left) and their average (black, right).

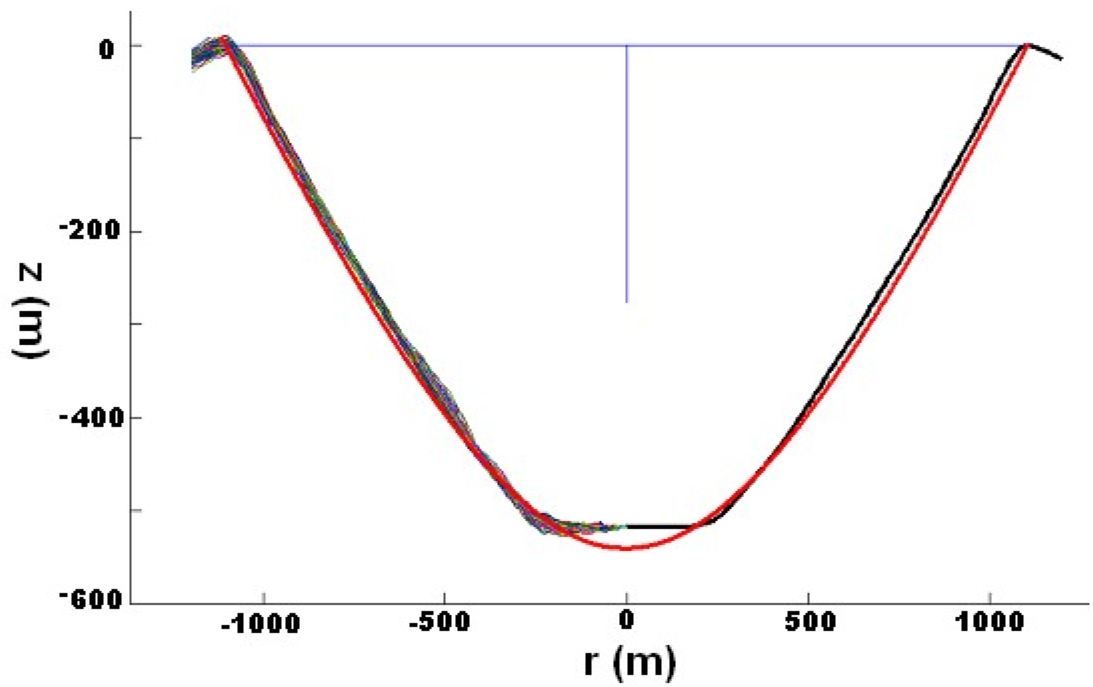

Fig.2b: The same as above, except vertically exaggerated ~3x , and the approximating hyperbola is plotted in red.