ANGULAR MOMENTA OF COLLIDED RAREFIED PREPLANETESIMALS. S. I. Ipatov [1,2,*], [1] Space Research Institute, Moscow, Russia; [2] Department of Physics, Catholic University of America, Washington DC, 20064, USA; [*] Present address: Alsubai Establishment for Scientific Studies, Doha, Qatar; siipatov@hotmail.com.

Introduction: In recent years, the formation of rarefied preplanetesimals - clumps was studied by several scientists (e.g. [1-4]). Ipatov [5-7] and Nesvorny et al. [8] supposed that trans-Neptunian binaries were formed from rarefied preplanetesimals (RPPs). Nesvorny et al. [8] calculated contraction of RPPs supposing that RPPs got their angular momenta when they formed from the protoplanet cloud. Nesvorny et al. noted that simulations made by Johansen et al. seem to generally indicated prograde rotation. Angular momenta of some observed trans-Neptunian binaries are negative. Ipatov [6] supposed that a considerable fraction of trans-Neptunian binaries could get the main part of their angular momenta due to collisions of RPPs. He showed that the angular momenta obtained at collisions of RPPs moving in circular heliocentric orbits can have the same values as the angular momenta of discovered trans-Neptunian and asteroid binaries.

Below the angular velocities used by Nesvorny et al. [8] as initial data are compared with the angular velocities obtained at a collision of two RPPs moving in circular orbits. Mergers of colliding preplanetesimals and formation of trans-Neptunian binaries are also discussed.

Angular momentum of two collided preplanetesimals: Ipatov [6] obtained that the angular momentum of two collided RPPs (with radii r_1 and r_2 and masses m_1 and m_2) moved in circular heliocentric orbits is $K_s=k_\Theta\cdot(G\cdot M_S)^{1/2}\cdot(r_1+r_2)^2\cdot m_1\cdot m_2\cdot(m_1+m_2)^{-1}\cdot a^{-3/2}$, where G is the gravitational constant, M_S is the mass of the Sun, and the difference in semimajor axes a of RPPs equals $\Theta\cdot(r_1+r_2)$. At $r_a=(r_1+r_2)/a<<\Theta$ and $r_a<<1$, one can obtain $k_\Theta\approx(1-1.5\cdot\Theta^2)$. The mean value of $|k_\Theta|$ equals 0.6. The resulting momentum is positive at $0<\Theta<(2/3)^{1/2}\approx0.8165$ and is negative at $0.8165<\Theta<1$. In the case of uniform distribution of Θ, the probability to get a reverse rotation at a single collision is $\approx1/5$.

The angular velocity ω of the RPP of radius $r=(r_1^3+r_2^3)^{1/3}$ formed as a result of a collision equals K_s/J_s, where $J_s=0.4\cdot\chi\cdot(m_1+m_2)\cdot r^2$ is the momentum of inertia of the RPP, and $\chi=1$ for a homogeneous sphere considered in [8]. Ipatov [6] obtained that

$\omega=2.5\cdot k_\Theta\cdot\chi^{-1}\cdot(r_1+r_2)^2\cdot r^{-2}\cdot m_1\cdot m_2\cdot(m_1+m_2)^{-2}\Omega$, where $\Omega=(G\cdot M_S)^{1/2}a^{-3/2}$ is the angular velocity of the motion of the RPP around the Sun. As $\omega=K_s/J_s$ is proportional to $(r_1+r_2)^2\cdot r^{-2}$, then ω does not depend on r_1, r_2, and r, if $(r_1+r_2)/r$=const. Therefore, ω will be the same at different values of k_r if we consider RPPs with radii $r_i=k_r\cdot r_{Hi}$, where $r_{Hi}\approx a\cdot m_i^{1/3}(3M_S)^{-1/3}$ is the radius of the Hill sphere of mass m_i (m_1, m_2, or m). If at some moment of time after a collision of uniform spheres with radii r_1 and r_2, the radius r_c of a compressed sphere equals $k_{rc}\cdot r$, then (at χ=1) the angular velocity of the compressed RPP is $\omega_c=\omega\cdot k_{rc}^{-2}$. Below we consider $r_1=r_2$, $r^3=2r_1^3$, $m_1=m_2=m/2$, and χ=1. In this case, we have $\omega\approx1.575k_\Theta\cdot\Omega$, e.g., $\omega\approx0.945\Omega$ at $k_\Theta=0.6$.

Nesvorny et al. [8] made computer simulations of the contraction of preplanetesimals in the trans-Neptunian region. They considered initial angular velocities of preplanetesimals equal to $\omega_o=k_\omega\Omega_o$, where $\Omega_o=(G\cdot m)^{1/2}r^{-3/2}$, k_Θ = 0.5, 0.75, 1, and 1.25. In most of their runs, $r=0.6r_H$, where r_H is the Hill radius for mass m. Also $r=0.4r_H$ and $r=0.8r_H$ were used. Note that $\Omega_o/\Omega=3^{1/2}(r_H/r)^{3/2}\approx1.73(r_H/r)^{3/2}$, e.g., $\Omega_o\approx1.73\Omega$ at $r=r_H$.

Considering $\omega=\omega_o$, in the case of a collision of Hill spheres, we have $k_\omega=1.25\cdot2^{1/3}3^{-1/2}k_\Theta\cdot\chi^{-1}\approx0.909k_\Theta\cdot\chi^{-1}$. This relationship shows that it is possible to obtain the values of $\omega=\omega_o$ corresponding to k_ω up to 0.909 at collisions of RPPs for $k_\Theta=\chi=1$. In the case of a collision of Hill spheres and the subsequent contraction of the RPP to radius r_c, the obtained angular velocity is $\omega_{rc}=\omega_H(r_H/r_c)^2$, where $\omega_H\approx1.575k_\Theta\Omega$. For this RPP with radius r_c, $\omega_o=(r_H/r_c)^{3/2}\Omega_{oH}$ (where Ω_{oH} is the value of Ω_o for the Hill radius) and ω_{rc}/ω_o is proportional to $(r_H/r_c)^{1/2}$. At $r_c=0.6r_H$, the collision of Hill spheres can produce K_s corresponding to k_ω up to $0.909\cdot0.6^{-1/2}\approx$ ≈1.17. Nesvorny et al. [8] obtained binaries or triples only at k_ω equal to 0.5 or 0.75 (not equal to 1 or 1.25). Therefore, one can coclude that the initial angular velocities of RPPs used in [8] can be obtained at collisions of RPPs. Note that the values of ω at the moment of a collision are the same at collisions at different values of k_r, but ω_{rc} is proportional to $(r/r_c)^2$ in the case of contraction of a RPP from r to r_c.

Frequency of collisions of preplanetesimals: The number of collisions of RPPs depends on the number of RPPs in the considered region, on their initial sizes, and on the time dependences of radii of collapsing RPPs. Cuzzi et al. [1] obtained the «sedimentation» timescale for RPPs to be roughly 30-300 orbit periods at 2.5 AU for 300 μm radius chondrules. Both smaller and greater times of contraction of RPPs were considered by other authors. According to Lyra et al. [4], the time of growth of Mars-size planetesimals from RPPs consisted of boulders took place in five orbits.

Let us consider a planar disk consisted of N identical RPPs with radii equal to their Hill radii r_{Ho} and masses $m_o=6\cdot10^{17}$ kg. The ratio of the distances from the Sun to the edges of the disk is supposed to be equal to a_{rat}=1.67 (e.g., for a disk from 30 to 50 AU).

For an object with mass $m_o=6\cdot10^{17}$ kg $\approx 10^{-7}M_E$ (where M_E is the mass of the Earth), e.g., for a solid object with diameter d=100 km and density $\rho\approx1.15$ g cm^{-3}, its Hill radius equals $r_{Ho}\approx4.6\cdot10^{-5}a$. For circular orbits separated by this Hill radius, the ratio of periods of motion of two RPPs around the Sun is about $1+1.5r_{Ha}\approx1+7\cdot10^{-5}$, where $r_{Ha}=r_{Ho}/a$. In this case, the angle with a vertex in the Sun between the directions to the two RPPs changes by $2\pi\cdot1.5\cdot r_{Ha}\cdot n_r\approx0.044$ radian during n_r=100 revolutions of RPPs around the Sun. For $N=10^7$ and $M_\Sigma=m_oN=M_E$, a RPP can collide with another RPP when their semimajor axes differ by not more than $2r_{Ho}$, i.e., the mean number of RPPs which can collide with the RPP is $\approx2N\cdot r_{Ha}(a_{rat}+1)/(a_{rat}-1)\approx$ $\approx10^7\cdot4.6\cdot10^{-5}\cdot3.3/0.67\approx2.3\cdot10^3$. The mean number N_c of collisions of the RPP during n_r revolutions around the Sun can be $\approx2\cdot(1.5r_{Ha}\cdot n_r)\cdot2N\cdot r_{Ha}(a_{rat}+1)/(a_{rat}-1)\approx$ $\approx2.3\cdot10^3\cdot1.4\cdot10^{-4}n_r\approx0.3n_r$. N_c is proportional to $N\cdot r_{Ha}^2$, i.e. to $N\cdot m_o^{2/3}$ and $M_\Sigma\cdot m_o^{-1/3}$. Therefore, for $N=10^5$ and d=1000 km (i.e., for $M_\Sigma=10M_E$), N_c is also $0.3n_r$. N_c is smaller by a factor of 4 if the semimajor axes of collided RPPs differ by not more than r_{Ho} (not $2r_{Ho}$ as above). Some collisions were tangent and did not result in a merger. RPPs contracted with time. Therefore, the real number of mergers can be much smaller than that for the above estimates. It may be possible that a greater fraction of RPPs had not collided with other RPPs before they contracted to solid bodies. In this case, RPPs contracted relatively quickly.

Mergers and contraction of preplanetesimals: Densities of RPPs can be very low, but their relative velocities v_{rel} at collisions were also very small. The velocities v_{rel} were smaller than the escape velocities on the edge of the Hill sphere of the primary [6]. If collided RPPs are much smaller than their Hill spheres, and if their heliocentric orbits are almost circular before the encounter, then the velocity of their collision does not differ much from the parabolic velocity v_{par} at the surface of the primary RPP (with radius r_{pc}). Indeed, v_{par} is proportional to $r_{pc}^{-0.5}$. Therefore, collisions of RPPs could result in a merger (followed by possible formation of satellites) at any $r_{pc}<r_H$ (if RPPs are rarefied). Johansen et al. [2] determined that the mean free path of a boulder inside a cluster - preplanetesimal is shorter than the size of the cluster. This result supports the picture of mergers of RPPs. In calculations made by Johansen et al. [3], collided RPPs merged. Given a primary of mass m_p and a much smaller secondary, both in circular heliocentric orbits, one can obtain that the ratio $v_\tau/v_{esc\text{-}pr}$ of tangential velocity of an encounter up to the Hill sphere to the escape velocity on the edge of the sphere is proportional to $m_p^{-1/3}\cdot a^{-1}$. Therefore, RPPs are more likely to merge when the primary is more massive and located farther from the Sun.

If a RPP got its angular momentum at a collision of two RPPs at a=1 AU, k_Θ=0.6, k_r=1, and $m_1=m_2$, then the period T_{s1} of rotation of the planetesimal of density ρ=1 g cm^{-3} formed from the RPP with intial radius $k_r\cdot r_H$ is ≈0.5 h, i.e., is smaller than the periods (3.3 and 2.3 h) at which velocity of a particle on a surface of a rotating spherical object at the equator is equal to the circular and escape velocities, respectively. T_{s1} is proportional to $a^{-1/2}\rho^{-2/3}$. Therefore, T_{s1} and the fraction of the mass of the RPP that could contract to a solid core are smaller for greater a. For those collided RPPs that were smaller than their Hill spheres and/or differ in masses, K_s was smaller (and T_{s1} was greater) than for the above case with k_r=1 and $m_1=m_2$.

Formation of trans-Neptunian binaries. The above discussions could explain why a larger fraction of binaries are found at greater distances from the Sun, and why the typical mass ratio (secondary to primary) is greater for trans-Neptunian objects than for asteroids. The binary fractions in the minor planet population are about 2% for main-belt asteroids, 22% for cold classical trans-Neptunian objects, and 5.5% for all other trans-Neptunian objects [9]. We suppose that the fraction of RPPs collided with other RPPs during their contraction can be about the fraction of small bodies with satellites, i.e., it can be about 1/4 in the trans-Neptunian belt, and it is smaller in the asteroid belt. In the considered model of binary formation, two colliding RPPs originate at almost the same distance from the Sun. This point agrees with the correlation between the colors of primaries and secondaries obtained by Benecchi et al. [10] for trans-Neptunian binaries.

Conclusions: The angular momenta of rarefied preplanetesimals needed for formation of small-body binaries can be obtained at collisions of preplanetesimals. Trans-Neptunian objects, including those with satellites, could be formed from contracting rarefied preplanetesimals.

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