CONSTRAINTS ON THE SIZE OF ASTEROID 216 KLEOPATRA USING INTERNAL STRESSES

Toshi Hirabayashi[1], Daniel J. Scheeres[1], Keith A. Holsapple[2], [1]University of Colorado at Boulder, CO 80309-0429, USA, [2] University of Washington, WA 98195-2400; Masatoshi.Hirabayashi@Colorado.edu

Abstract: The size of Asteroid 216 Kleopatra is constrained by combining information on its shape and mass along with an internal stress failure analysis. This asteroid has likely undergone plastic deformation in its past and is likely on its failure envelope currently.

Introduction: The main belt Asteroid 216 Kleopatra was observed using radar by Ostro et al. [1], from which a detailed shape model was estimated. The mean radius of the asteroid was reported to be 54.3 km, although its shape is significantly distended as seen in Fig. 1. The uncertainty in the total size of the asteroid was stated to be as large as 25%. The surface bulk density of the asteroid was estimated to be 3.5 $\mathrm{g/cm^3}$ based on surface reflectivity.

In Descamps et al. [2] a series of optical and adaptive optics (AO) observations of the asteroid are reported, along with the discovery of two satellites. Based on the orbital period of the satellites they determine a precise mass estimate for the asteroid, 4.64×10^{18} kg. Their AO observations confirm the main features of the Ostro et al. shape model, however they indicate a somewhat larger total size for the body, at a factor of 1.23 times larger than the Ostro et al. size.

Combining the Ostro et al. size and the Descamps et al. mass we find a bulk density of around 6.5 $\mathrm{g/cm^3}$, about twice the estimated surface bulk density from the radar observations. If we assume that the size of the asteroid is uniformly 1.23 times larger, based on the Descamps et al. analysis, the bulk density of the asteroid decreases to 3.5 $\mathrm{g/cm^3}$, which is a large enough difference to have significant implications for the structure and stability of the asteroid shape. In fact, combined with the rapid spin of this asteroid, we find that significant constraints can be placed on the geophysical properties of this asteroid as a function of its size and hence its density. To do this we use the Mohr-Coloumb yield criterion as a function of the internal friction angle and the cohesion for stress solutions based on the actual shape of this asteroid. We make a uniform density distribution assumption for the current work, but plan to relax this in the future.

Physical Parameters of Asteroid 216 Kleopatra: In the following we summarize the physical parameters used in this abstract. First, we use the three dimensional shape model, whose equivalent radius is 54.3 km, obtained by Ostro et al. [1] based on radar observation. To investigate the different proposed sizes of this asteroid we uniformly change the scale of this shape model by a factor ranging from 1 to 1.5. Figure 1 shows the projection of Asteroid 216 Kleopatra onto the $x-y$ and $x-z$ plane whose axes are aligned along the principal axes (x, y, and z are the minimal, intermediate, and maximum principal axis, respectively.). For the analysis we fix the mass estimated by Descamps et al. [2] and assume a uniform density. Combined with our range of size scales, this corresponds to a bulk density of the asteroid ranging from 6.5 to 1.9 $\mathrm{g/cm^3}$ as the size scale is changed from 1.0 to 1.5, respectively. The asteroid is assumed to be spinning about its maximum moment of inertia with a spin period of 5.385 hour [1, 2]. The cohesion of Asteroid 216 Kleopatra is assumed to be zero, as the size of this asteroid is large enough for cohesive forces to be negligible. However, the internal friction angle is a significant parameter and is not known, thus we use this quantity as a changeable parameter.

Analysis Method: There are two free parameters in our study, the size scale and the internal friction angle. In addition to these we also enforce a constraint that the interior of the asteroid, or some appropriately averaged section of it, should be within the Mohr-Columb yield criterion. To evaluate the MC criterion we compute interior stresses by three different techniques: an elastic solution using a commercial finite element method (ANSYS), a classical total-volume average, and a new partial-volume average technique developed for this study. At a given asteroid size scale we then determine the maximum friction angle for which the interior of the body is within the yield envelope (see Fig. 2). We assume that the material is elastic-perfect plastic.

For the finite element method, we break the Kleopatra shape model into a fine mesh and use the standard engineering analysis tool ANSYS to estimate the elastic stress field (assuming a Youngs modulus of 1×10^6 Pa and a Poissons ratio of 0.2). After computing this stress, we evaluate the equivalent stress defined by the Mohr-Coulomb yield condition. This solution allows us to compute the minimum friction angle for which no interior points experience the first yield and to identify regions within the body which are the most likely to deform plastically first.

At the other extreme, the total-volume-averaged stress is computed by taking the average of the body forces over the whole body (using the same mesh as above). This allows us to find a lower bound on the friction angle for which the entire body will start to fail. This approach does not rely on finding an elastic solution for the interior stresses, as is discussed in detail by Holsapple [5] and Chen and Han [6].

The partial-volume-averaged stress is obtained in a similar way as the total-volume-averaged stress analysis, but is only computed over a distinct part of the body. Our method specifically considers a slice of the asteroid and computes the average forces across this slice and incorporates surface forces where the remaining sections of the asteroid press on the slice under consideration. Focusing on those regions of the asteroid that are most likely to be at or on the failure envelope, this allows us to compute the lower friction angle for collapse of a specific part of the asteroid. The partial-volume-averaged stress is a new technique to see whether or not specific regions of the asteroid may fail. For this study, we only use this stress analysis over a small cross section region perpendicular to the minimum principal axis to evaluate the body failure conditions when the plastic region propagates across the cross section.

Result: In Fig.2, the red line is the friction angle for the total-volume average, the purple line is that for the first yield condition, and the blue line is that for the partial-volume average. For the first yield condition the friction angle stays almost constant around 63 degree in the size scale ranging from 1.0 to 1.24, but increases dramatically when the size scale is larger than 1.24. In the case of the total-volume average, the friction angle keeps less than 20 degree in the size scale from 1.0 to 1.38 and increases gradually when the size scale is larger than 1.38. Also, it is found that when the size scale is 1.24, there is a minimum value of about 1 degree. To determine the partial-volume average we choose a cross section perpendicular to the minimum principal axis, whose minimum section is located at $x = 0.1x_{min}$ and maximum section is at $x = 0.3x_{max}$ (x_{min} and x_{max} are the minimum and maximum point of the shape, respectively). For the Kleopatra shape scaled by a factor 1.23 this corresponds to the section ranging from $x = -10$ km to $x = 40$ km. The friction angle is 43 degrees for the size scale of 1.0, while it is 17 degrees for a size scale of 1.28. When the size scale is larger than 1.28, the friction angle increases dramatically.

Discussion: Note that Scott [4] showed that the friction angle for high porosity sand ($\sim$40%) is around 35^o (See Fig.7-5(a) on p.309). Assuming this value as an analog for the asteroid interior, the fact that the first yield condition gives very high friction angles (compared to [4]) implies that the elements that violate the yield condition may have deformed from the elastic solution to be on or below the yield condition. The regions that control this high friction angle are within the neck portion of Kleopatra. Thus we perform our partial-volume-averaging across this interface in order to ascertain the friction angle required for this entire region to be stable independent of the elastic solution. Compared to the value of friction angles by Scott [4], the partial-volume average implies that a size scale of 1.14 to 1.32 may be an allowable range for the asteroid. Perhaps most profound, these results imply that internal regions of Asteroid 216 Kleopatra are on or close to the failure envelope and thus may have plastically deformed to achieve the current shape of the body. Future astronomical observations should be able to better measure the total size of this asteroid, which in turn will provide more constraints on our analysis of this unique body.

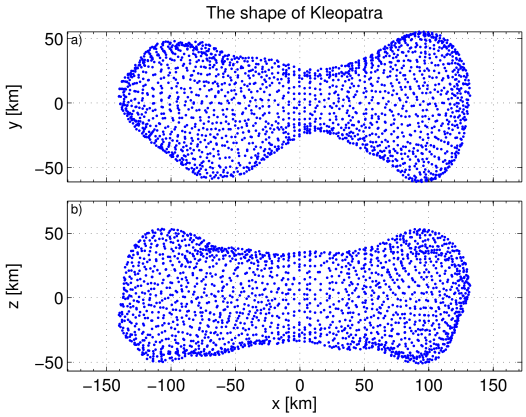

Figure 1: Kleopatra projected onto the $x - y$ plane (Upper figure) and the $x - z$ plane(Lower figure). The shape has been scaled by a factor of 1.23.

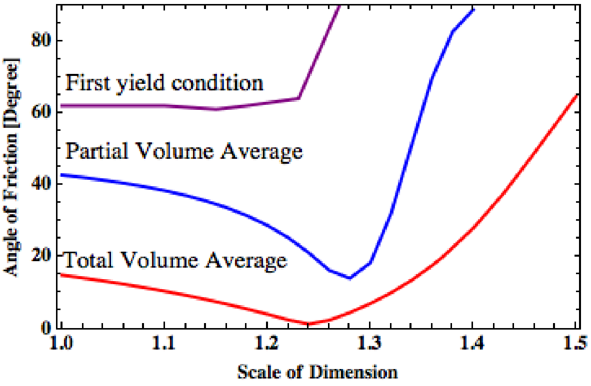

Figure 2: Minimum friction angles obtained by the three different computations. The red line shows results from the total-volume average, the purple line from the elastic solution using ANSYS, and the blue line from the partial-volume average method.

References: [1] S. J. Ostro, et al. (2000) *Science* 288:836. [2] P. Descamps, et al. (2011) *Icarus* 211:1022. [3] K. A. Holsapple (2007) *Icarus* 187:500. [4] R. F. Scott (1963) *Principles of Soil Mechanics* Addison-Wesley. [5] K. A. Holsapple (2008) *International journal of Non-Linear Mechanics* 43:733. [6] W. F. Chen, et al. (1988) *Plasticity for Structural Engineers* Springer-Verlag.