

WIND DRIVEN CAPILLARY-GRAVITY WAVES ON TITAN: HARD TO DETECT OR NON-EXISTENT?

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Summary: Saturn's moon Titan supports standing bodies of liquid under a dense atmosphere. On Earth, it is rare to observe a body of water whose surface is not disturbed by some form of wave activity. On Titan, Cassini spacecraft observations thus far show no indication of surface waves in Titan's hydrocarbon lakes [1]. This observation is intriguing given the predominance of aeolian features at equatorial latitudes, which require winds capable of saltating sand sized particles.

Here, we investigate analytically the conditions necessary for the onset of exponential growth in capillary gravity wave amplitude in Titan's lakes using modern theories of wind wave generation. This work differs from previous studies by simultaneously accounting for the gravity, viscosity, surface tension, and air/liquid density relevant to the Titan environment. (Table 1). For liquid compositions varying between pure methane and equilibrium mixtures with the atmosphere, we find that the threshold wind speed for wave generation is 2.5-4.5 times lower than on sea-water on Earth at 30C, depending primarily on liquid viscosity.

While polar equinoxial winds are predicted to have been calm, consistent with the observed absence of wave activity, winds speeds are predicted to increase during northern spring and summer [2] (Figure 3). This more lively wind regime, which will be investigated during the Cassini Solstice Mission, is predicted to produce wind speeds capable of exceeding the modeled thresholds and potentially exciting wind waves. The time and frequency of wave activity (or lack thereof) observed during the Solstice Mission may provide constraints on liquid composition through the viscosity dependence of threshold winds.

Table 1: Model Parameters.

Parameter	Earth	Titan
ϵ_{liq}	53.98+34.38i	1.8+0.0007i
ρ_{air}	0.0012 g/cm ³	0.005 g/cm ³
ρ_{liq}	1 g/cm ³	0.6 g/cm ³
ν_{air}	0.154 cm ² /s	0.0126 cm ² /s
ν_{liq}	0.01 cm ² /s	0.005-0.03 cm ² /s
g	981 cm/s ²	135 cm/s ²
T	73 dynes/cm	18 dynes/cm
λ_{crit}	1.7 cm	3.0 cm
λ_{obs}	2.16 cm	2.16 cm

Generating Capillary-Gravity Waves: Wind blowing over a liquid surface generates waves that grow in amplitude with increasing wind speed. Energy can be transferred from the wind to the waves by either pressure fluctuations or tangential stresses. [3] developed a theory for wave generation, known as the resonance mechanism, where waves are initiated and grow in resonance with turbulent air pressure fluctuations (Figure 1). Resonance is only possible if the waves are traveling along with the turbulence, leading to a minimum wind speed necessary for excitation that is correlated to the minimum phase speed of the waves. Since the turbulent pressure fluctuations are in random phase with respect to the surface waves, the resonance mechanism can only account for linear growth (similar to a random walk). Resonance waves are small (10's of micrometers amplitude) and are not responsible for the observable disturbances associated with wind waves.

Once initial wavelets are generated their presence can modify the air flow in the boundary layer above them, generating additional pressure fluctuations (Figure 1). These wave-induced pressure fluctuations are in-phase with the topographic wave slope and can lead to exponential growth. [4] rigorously studied the possibility of energy transfer between the mean air flow and a wavy liquid surface, obtaining estimates for the wind speed necessary to trigger the onset of exponential growth that were in general agreement with field observations. Recently, the threshold associated with the onset of exponential growth has been observed experimentally by [5], providing support for the concept of a positive feedback mechanism between growing surface waves and airflow in the boundary layer.

Following the results of [5], we use the theory of [6] (a modern variant of [4]) to estimate the wind wave threshold for liquid hydrocarbon on Titan. The threshold is derived by balancing exponential wave growth (using the equations presented in [6] under Titan-relevant environmental conditions, see [7] for details) with loss due to viscous dissipation. For viscosities ranging between pure methane ($\nu=0.0031$ cm²/s) and the more complex equilibrium compositions predicted by [8] ($\nu=0.026$ cm²/s), U_{10}^{th} ranges between 0.4 and 0.7 m/s. For pure ethane at 94K ($\nu=0.017$ cm²/s), $U_{10}^{th} \sim 0.6$ m/s. Figure 2 shows the threshold wind speed as a function initial wavelength and liquid viscosity (i.e., liquid composition).

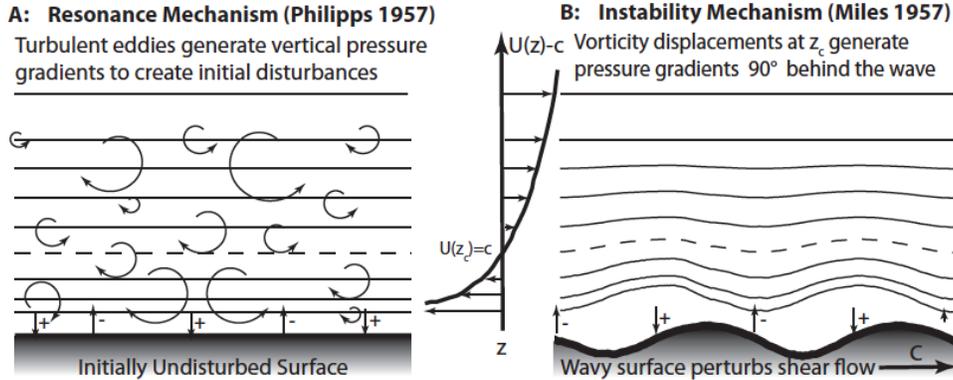
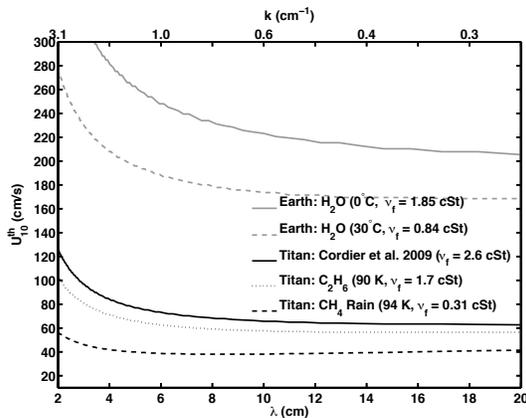


Figure 1: Physical depiction of how the resonance [3] and instability [4] mechanisms excite wind waves. For a detailed review see [8,9]. **A:** Turbulent eddies are advected over the liquid surface. If the horizontal scale of the associated vertical pressure gradients are similar to surface waves with phase speeds equal to the advection rate, the waves will linearly grow. **B:** In the critical layer ($U(z_c)=c$), local concentrations of excess vorticity peak over the nodes of the surface waves that are perturbing the shear flow. This vorticity field induces a velocity field which redistributes the vorticity field, which ... Feedback between these induces fields lead to a pressure component with a $\pi/2$ phase lag relative to the waves (exponential growth). **B** is adapted from [9].



For a detailed description of this work see [7].

References: [1] Soderblom, J.M. et al (2012) *Icarus* 220, 744-751. [2] Schneider, T. et al (2012) *Nature* 481 [3] Philipps O.M. (1957) *JFM*, 2 [4] Miles, J.W. (1957) *JFM*, 3, L5202 [5] Donelan, M.A. and W.J. Plant (2009), *JGR*, 114 C07012 [6] Donelan, M.A. and W.J. Pierson (1987) *JGR*, 92, 4971-5030. [7] Hayes et al. (2013) *Icarus*, in-press, 38 [8] Kinsman, B. (1984), *Prentice Hall* (ISBN 0486646521) [9] Lighthill, M.J. (1962) *JFM* 14, 385-398.

Figure 2: Threshold wind speed at 10 m (U_{10}^{th}) where the exponential growth rate of capillary gravity waves is balanced by viscous dissipation.

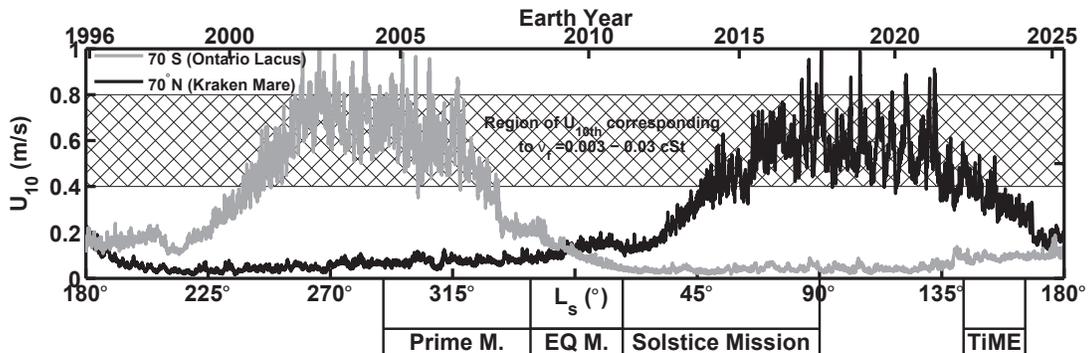


Figure 3: Wind speeds averaged over 24 hr at 10 m from [3] using a Titan GCM. Values represent the 95% quantile of the longitudinal distribution at 70S and 70N for a given time of year. The cross-hatched region highlights the expected range of threshold wind speeds for $\nu_i = 0.003 - 0.03$ cm²/s. Wind speeds are expected to exceed threshold during the spring/summer but remain below even the minimum threshold during each pole's respective equinox and winter. The L_s ranges of the Cassini Prime, Equinox, and Solstice Missions, as well as the recently proposed TiME mission are shown.