SIZE DEPENDENCE OF COEFFICIENT OF RESTITUTION: SMALL-SCALE EXPERIMENTS AND THE EFFECTS OF ROTATION. D. D. Durda[1], D. C. Richardson[2], E. Asphaug[3], and N. Movshovitz[3], [1]Southwest Research Institute, 1050 Walnut Street Suite 300 Boulder CO 80302 durda@boulder.swri.edu, [2]University of Maryland, College Park MD 20742, [3]University of California Santa Cruz, Santa Cruz CA 95064.

Introduction: Knowledge of the range of values of the coefficient of restitution (defined as the ratio of the relative speed of separation, v', to the relative speed of impact, v) and its dependence on the size/mass and material properties of the colliding bodies and the impact speed is critical for a number of problems in planetary studies, including planetary rings [1], asteroid collisons [2,3], and the accretion of planetesimals [4,5]. In the general case of nonelastic collisions, the energy loss E is related to the coefficient of restitution by

$$E = ½\mu v^2 (1 - \varepsilon^2), \quad (1)$$

where $\mu = m_1m_2 / (m_1 + m_2)$ is the reduced mass and $v = |v_2 - v_1|$ is the relative speed between the two bodies before the collision. For perfectly elastic collisions $\varepsilon = 1$; in the case of perfectly inelastic collisions, the two bodies stick together after the impact and $\varepsilon = 0$. For real bodies the numerical value of ε lies somewhere between 0 and 1 and depends primarily on the composition and physical properties of the colliding bodies, but is also a function of their relative impact speed and size/mass as well.

We have previously reported results of a series of large-scale experiments to measure the coefficient of restitution for 1-m-diameter rocky bodies in impacts with collision speeds up to ~1.5 m s^{-1} [6]. The experiments were conducted in an outdoor setting, with two 40-ton cranes used to suspend the ~1300-kg granite spheres pendulum-style in mutual contact at the bottoms of their respective paths of motion. The spheres were displaced up to ~1 meter from their rest positions and allowed to impact each other in normal incidence collisions at relative speeds up to ~1.5 m s^{-1}. Video data from 66 normal incidence impacts suggest a value for the coefficient of restitution of 0.83 ± 0.06 for collisions between ~1-meter-scale spheres at speeds of order 1 m s^{-1}. No clear trend of coefficient of restitution with impact speed is discernable in the data for the 1-meter granite spheres.

Small-Scale Experiments: In order to reveal any size dependence of the coefficient of restitution for the granite used in our previous large-scale experiments we need to gather data at smaller size scales *with the same granite*. The large granite spheres provided the stock material from which to mill the required smaller spheres. Fragments from the large spheres, gathered after a series of cratering experiments on them [7], were milled into ~1-inch-diameter spheres. We have also collected two additional rock types for which to gather small-scale coefficient of restitution data: a massive basalt collected from a recent lava flow near Flagstaff, AZ, and a aeolian sandstone from the Lyons Formation near Boulder, CO.

We are gathering new coefficient of restitution data for these smaller-scale rock spheres following the methods described by [8]. Imre et al. [8] determined the coefficient of restitution for low-speed rock-on-rock impacts by dropping ~cm-scale rocky spheres onto flat rock slabs and measuring the ratio of flight times during subsequent bounces. As defined above, the coefficient of restitution can be expressed as the ratio between the speed of separation v_{n+1} and the speed of approach v_n as the sphere bounces to rest:

$$\varepsilon = |v_{n+1}| / |v_n| \text{ (n=1,2,3,…)}. \quad (2)$$

For impacts represented by the bounce of a ball on a half-space under gravity g, the flight time from the top of the trajectory to the surface is one-half of the total flight time T_n between the nth and (n+1)th bounces:

$$v_n = ½gT_n \text{ (n=1,2,3,…) [m/s]}. \quad (3)$$

It is then valid to substitute the speed terms of Eq. (2) by Eq. (3) for the nth and (n+1)th bounces. The coefficient of restitution can now be expressed as the ratio between the total flight times T_{n+1} and T_n:

$$\varepsilon = T_{n+1} / T_n \text{ (n=1,2,3,…)}. \quad (4)$$

To measure the coefficient of restitution, only measurements of the flight times T_n between the bounces are now required to solve this equation.

Flight times are determined by recording the sound of the bounce sequences using a Zoom H4n recorder with 96 kHz/24-bit sampling. The recorded sound files are imported into Adobe Audition and the difference in times between sequential bounce sound pulses is measured (Fig. 1).

Preliminary tests dropping ¼-inch plastic and aluminum BBs onto a slab of basalt from a height of ~1m yielded cofficients of resitution of ~0.85 and ~0.61, respectively. Initial 'production' experiment runs with basalt-on-basalt impacts from drop heights of 6.4, 9.3, and 11.3 cm (giving first-bounce impact speeds of 1.120, 1.350, and 1.488 m/s, respectively) yield coefficients of restitution of 0.831, 0.843, and 0.794, repsectively. These values are in the same range that we determined for our ~1-m granite spheres and that are quoted in [8]. The full suite of experiment runs for all three rock types is in progress and results will be reported at the meeting.

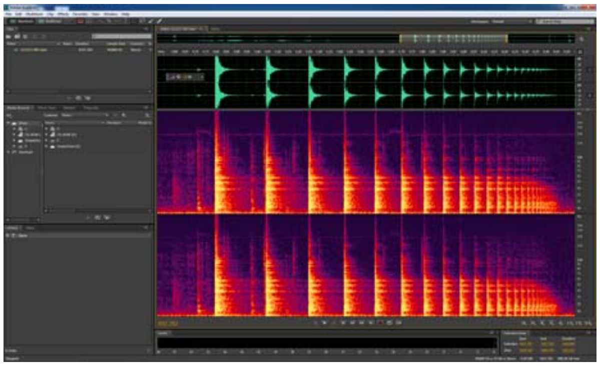

Figure 1: An example of a recorded bounce sound sequence displayed in Adobe Audition. Bounce times can be determined to ~0.00001 s precision.

Effects of Rotation in Large-Scale Experiments: Our large-scale crane operations in May 2009 [6] were so successful that by the end of the third day of pendulum experiment data taking we had already achieved the full range of impact conditions we had proposed. With another full day of crane operations available to us we quickly composed a series of impact experiments to examine the effects of body rotation on the coefficient of restitution. The challenge has been to develop a data analysis and reduction pipeline for data that we had not previously planned to gather and for which we had little time to 'think through' from a reduction and analysis perspective. The data are video image sequences from about ten supplemental low-speed collision experiments; the fundamental quantities that must be extracted from the video frames are the linear translation speeds and angular rotation rates of the rotating and colliding large granite spheres. Fortunately, a number of commercial software packages used in the visual effects industry has led to a very workable solution.

SynthEyes™ is a camera tracking (a.k.a. match-moving) and image stabilization software application routinely used to composite live action and digital effects into seamless visual effects sequences for Hollywood movies. A key feature of SynthEyes is its ability to identify trackable elements in video sequences, follow those elements from frame to frame, and then formulate a full 3D camera tracking solution.

Initial tests applied to our video files have proven SynthEyes to be very successful in producing good tracking solutions. The SynthEyes solutions are then imported into Modo, a 3D modeling software package that readily shows the XYZ coordinates and axis orientations of the SynthEys-solved sphere centers of mass.

The basic workflow we have identified from our initial tests proceeds roughly as follows. Video from calibration spin tests provides the initial FOV for the camera when fully zoomed in, and helps create a scale 'model' of each of the spheres. 3D models with simplified surface geometry are derived for each sphere; the models are to real world scale (1-meter-diameter spheres). When solving for the full 3D geometry of each of the experiment runs this geometry is imported and the various tracks corresponding to each of the visible surface points is locked to the actual model, forcing the solution to proper scale and orientation simultaneously.

Figure 2 shows a SynthEyes tracker solution for one of our experiment runs as a test of the workflow described here. The distance to the camera in the spin tests solves independently to about 23.65m, very close to our measurement of 24m made during the actual experiments.

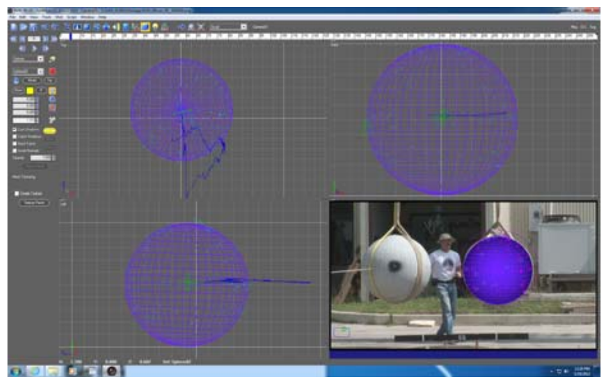

Figure 2: SynthEyes tracker test of collision experiment run 99, in which an initially non-rotating impacting sphere collides with an initially stationary but rotating sphere.

Once SynthEyes has obtained a good camera tracking solution, that camera solution can be 'inverted' to instead represent a full 3D solution for the translation and rotation of the center of the point cloud of surface tracker points on each sphere, thereby giving the necessary quantities for solving for the normal and tangential coefficients of restitution and allowing a determination of angular momentum transfer efficiency in such impacts.

References: [1] Porco C.C. et al. (2008) *Astron. J.* **136**, 2172–2200. [2] Durda D.D. et al. (2004) *Icarus* 170, 243-257. [3] Michel P., et al. (2002) *Icarus* **160**, 10–23. [4] Richardson et al. (2009) Plan. & Space Sci. 57, 183–192. [5] Leinhardt Z.M. et al. (2009) *Mon. Not. R. Astron. Soc.* **396**, 718–728. [6] Durda D.D. et al. (2011) *Icarus* **211**, 849–855. [7] Walker et al. (2012) http://www.sciencedirect.com/science/article/pii/S0734743X12001558. [8] Imre et al. (2008) *Comp. & Geosci.* **34**, 339–350.