TIDAL RESPONSE OF A LATERALLY VARYING MOON: AN APPLICATION OF PERTURBATION THEORY.

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Introduction: The Moon displays a number of hemispherically asymmetric features. 1) The farside has a much higher surface elevation than the nearside, suggesting that the farside has a thicker crust [1]. 2) The mare basalts, formed by the most intense volcanism in lunar geological history, erupted predominantly on the nearside from ~3.9 Ga to ~3 Ga [2]. 3) Deep moonquakes (DMQs), detected by Apollo seismic stations, are located mostly on the nearside at depths of ~800 km [3].

Crustal production and the mare basalts eruption are consequences of the early thermochemical evolution of the Moon. The hemispherical asymmetry of the crustal thickness and mare basalts suggests long-wavelength lateral variability in the early Moon's thermochemical structure [4,5,6]. The DMQ distribution reflects the present-day state of the lunar interior. A statistically established correlation between the locations of DMQs' epicenters and the mare basalts distribution suggests that the long-wavelength structure of the early Moon is likely still a feature of the present-day lunar mantle [7].

The recently completed GRAIL (Gravity Recovery and Interior Laboratory) observations provide lunar gravity field data to unprecedented precision (<1% at long wavelengths) [8]. In a previous publication, we proposed to use GRAIL observations to constrain the Moon's internal structure, by solving for tidal variations in the Moon's gravity field, searching specifically for spherical harmonic tidal terms that would result from lateral variations in elastic moduli [9]. We numerically determined the tidal response of a Moon with laterally varying mantle structure. However, a thorough interpretation of the numerical results, particularly the mode coupling effects, was not included [9]. To understand our numerical results from a theoretical point of view, we here develop an analytic method based on perturbation theory [10] to complement the numerical approach.

Physical Model and Methods: The lunar tidal force is predominantly at spherical harmonic degree 2 (i.e. l=2, m=0, and l=2, m=2, where l and m are spherical harmonic degree and order) [11]. When such a tidal force is applied to a spherically symmetric planetary body, the dynamic response of the body must have the same spatial pattern, with the same corresponding degree and order. However, if lateral variations in elastic moduli (i.e. shear wave speed) of the planetary body exist, the tidal response will occur not only at degree-2 but also at other spherical harmonic degrees, with amplitudes depending on the wavelengths and amplitudes of the structural variations.

For a spherically symmetric body, the response to the tidal force is expressed in terms of the Love numbers h and k (for the radial displacement and gravitational potential, respectively). To determine the non-degree-2 response, we first resort to numerical methods by using the finite element code CitcomSVE that was originally designed to solve the post-glacial rebound problem for the Earth with a 3-D viscoelastic incompressible mantle [12, 13], and later modified to include compressibility [14]. For this study, we compute the tidal response of a Moon that has degree-1 laterally varying mantle structure.

Our numerical results show that a degree-2 tidal force acting on a Moon with degree-1 structure will give rise to a non-degree-2 tidal response, which can be interpreted in terms of mode coupling effects [9]. To understand the mechanism of the coupling and the behavior of the non-degree-2 spherical harmonic terms in the response, we develop a semi-analytic method based on perturbation theory [10]. We treat the degree-1 lateral variation in lunar mantle structure as a relatively small structural perturbation to a spherically symmetric background. A degree-2 tidal force acting on a spherically symmetric mantle gives rise to a zeroth order dynamic response at the same degree and order (i.e., degree 2). According to perturbation theory, this zeroth order degree-2 response couples with the degree-1 structural variations and excites first order non-degree-2 terms in the response. Those first order non-degree-2 terms continue to couple with the same degree-1 structure, introducing second order terms at additional degrees and orders. In principle, such modifications to the tidal response can be computed to any order in a semi-analytic way.

Results and Discussions: We use the finite element code CitcomSVE to compute the tidal response of a Moon with degree-1 order-1 (l=1, m=1) lateral variations in mantle shear modulus μ, expressed in the form of dV_s/V_s, which is the relative variation of the shear wave speed ($V_s = \sqrt{\mu / \rho}$, where ρ is the mantle density). The amplitude of the variation is a variable. We choose this l=1, m=1 spatial pattern because it best represents the hemispherical asymmetry of the lunar mantle. The tidal response is expanded into spherical harmonics with degrees up to l=10, and is expressed in terms of relative amplitudes at each degree and order. The most significant non-degree-2 responses are at degree 3: (*l*=3, *m*=1) for a (*l*=2, *m*=0) tidal force, and (*l*=3, *m*=1; *l*=3, *m*=3) for a (*l*=2, *m*=2) tidal force. In addition, the degree-3 responses are found to grow linearly with an increasing amplitude of dV_S/V_S, as shown in Figure 1.

Using identical mantle properties and the same definition of the degree-1 structural variation, we compute a semi-analytic solution for the tidal response to first order in the perturbation. Our first order semi-analytic solution successfully predicts many features of the CitcomSVE solution. 1) The harmonics predicted by the first order solution are exactly the same as those that dominate the CitcomSVE results: (*l*=3, *m*=1) for a (*l*=2, *m*=0) tidal force and (*l*=3, *m*=1; *l*=3, *m*=3) for a (*l*=2, *m*=2) tidal force. 2) The first order semi-analytic solutions predict that the amplitudes of the degree-3 responses are proportional to the amplitude of the shear wave speed variation, and thus explain the linear dependence of the degree-3 responses on the amplitude of dV_S/V_S. 3) The semi-analytic results predict specific ratios between amplitudes of the degree-3 responses, of $\sqrt{12}:-1:\sqrt{15}$, between the (*l*=3, *m*=1) response to the (*l*=2, *m*=0) tidal force, the (*l*=3, *m*=1) response to the (*l*=2, *m*=2) tidal force, and the (*l*=3, *m*=3) to the (*l*=2, *m*=2) tidal force, respectively. This is in near-perfect agreement with the numerical results. 4) The semi-analytic solutions for the first order degree-3 responses predict the numerical results reasonably well, with relative differences of ~10%, as shown in Figure 1.

The success of the perturbation theory to first order suggests it might be a promising tool for understanding mode coupling effects at higher orders of perturbation. Our second order analytic analysis predicts second order tidal responses at modes of (*l*=2, *m*=0; *l*=2, *m*=2; *l*=4, *m*=0; *l*=4, *m*=2) for (*l*=2, *m*=0) tidal forcing, and of (*l*=2, *m*=0; *l*=2, *m*=2; *l*=4, *m*=0; *l*=4, *m*=2; *l*=4, *m*=4) for (*l*=2, *m*=2) tidal forcing, and it predicts that these responses grow quadratically with the amplitude of the structural variation. Figure 1 displays these components of the CitcomSVE response. With the exception of a few results for small amplitude perturbations, this expected quadratic dependence on the amplitude of dV_S/V_S is clearly evident in the CitcomSVE. We will continue to explore the second order semi-analytical solutions and present the characteristics of the second order tidal response. We expect the combination of numerical and analytical analyses will provide better insight into tidal loading physics and an improved understanding of how GRAIL tidal results can be used to constrain the interior structure of the Moon.

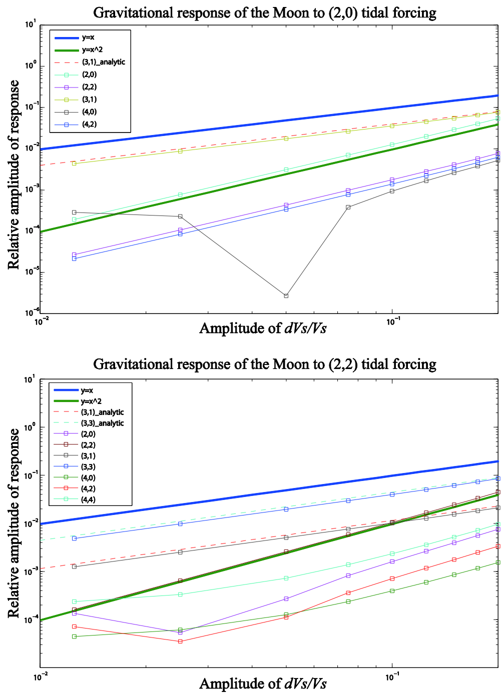

Figure 1. Amplitudes of spherical harmonic terms in the tidal response of the Moon's gravitational potential, vs. the amplitude of dV_S/V_S in a log-log scale. The top and bottom plots are the responses to (*l*=2, *m*=0) and (*l*=2, *m*=2) tidal forcing, respectively. The marked solid lines are CitcomSVE results at the harmonics predicted by first-order and second-order semi-analytic results. The dashed lines are the first order semi-analytic predictions at degree 3; they show only small relative differences with the degree-3 responses from CitcomSVE. The widened solid lines show linear and quadratic forms in log-log scale.

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