USING ELASTIC TORQUE TO PREDICT LIBRATION ON ICY SATELLITES

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Introduction: Rotating objects in the solar system will experience different forms of variation in rotation rate depending on the structure of the body. One known modification of spin rate which acts on rigid bodies is driven by gravitational torques produced by the geometric libration of an elliptic orbit. This idea has been applied to the icy satellites of Jupiter, in particular Europa, with the aim of predicting their librations [1]. We, however, expect a different form of torque will drive the rotation rate variation of bodies having an elastic ice shell surrounding a fluid subsurface ocean layer, such as the Galilean satellites. Using our method to calculate elastic libration amplitude for ice shells surrounding a fluid layer, we find that the value is dependent on the ratio of "Effective Elastic Thickness" to total ice shell thickness. This elastic thickness is dependent on the rheology and layering of the ice shell and therefore, different shell structures can predict the same libration amplitude.

Elastic Restoring: Our previous work has been focused on deriving the response of an elastic ice shell forced by an eccentric orbit. Initially we found that the elastic energy stored in an ice shell due to a reorientation of that shell is proportional to the angle of reorientation (α) squared. Therefore,

$$\frac{\delta E(\alpha)}{\delta\alpha} = \tau(\alpha) = -k\alpha, \tag{1}$$

where k describes the proportionality between energy and reorientation angle squared. Applying this elastic restoring torque to orbital timescales and using $\alpha = \lambda - 2esin(nt)$ we find that the shell will librate about the system's guiding center at the orbital frequency with an amplitude of

$$\lambda(t) = \frac{2e\omega_0^2}{\omega_0^2 - n^2}, \tag{2}$$

where e is the body's orbital eccentricity, n is its mean motion, and ω_0 is the free libration frequency of the body. This quantity, ω_0 has the value $\omega_0^2 = \frac{k}{C}$, where C is the spin axis moment of inertia of the ice shell and k is the proportionality constant described above. This results in a libration amplitude having the form,

$$\lambda(t) = \frac{2e}{1 - \frac{Cn^2}{k}}. \tag{3}$$

Multi-layered Viscoelastic Response: We extend the idea of elastic restoring by introducing additional lower ice layers and incorporating a viscous element to those layers.

Degree of Elasticity. One can determine the degree of elasticity of an object based on the Delta parameter, given by

$$\Delta = \frac{\mu}{\eta\omega}, \tag{4}$$

where ω, η, and μ are the forcing frequency, viscosity, and shear modulus respectively. Materials with a $\Delta \gg 1$ behave as fluids, while those with $\Delta \ll 1$ behave elastically. In the transition state, materials having $\Delta \approx 1$ are termed viscoelastic. The forcing we wish to consider is given by the tides at the orbital frequency, n. Together with a shear modulus for ice of 3.487×10^9 Pa, we can determine which regime an ice shell layer will be in given its viscosity. Fig. 1 demonstrates these possible states for Europa. Red lines indicate approximate regions of elastic, viscoelastic, and fluid conditions. The green lines highlight approximate viscosities that correspond to these material conditions.

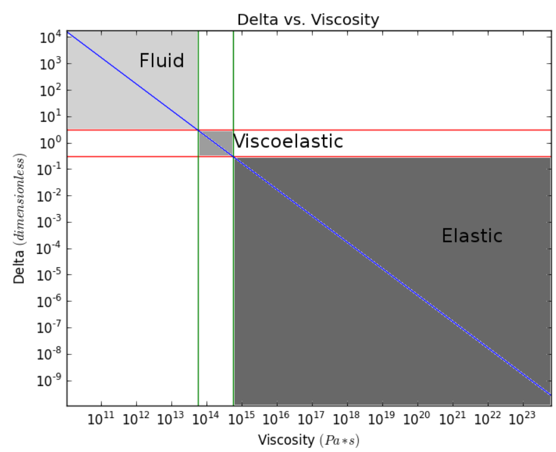

Figure 1: Viscoelastic regimes on Europa over a range of plausible ice viscosities.

Estimates for surface and upper ice viscosity range up to $\eta \approx 10^{21}$ Pa s [2], well within the elastic region. The viscosity is thought to drop off considerably with depth into the ice as temperatures increase toward the warmer ice-ocean interface. Some estimates for lower layer viscosities go as low as $\eta \approx 10^{12}$ Pa s [2], well within the fluid region. Therefore, the shell transitions from elastic to fluid with depth at the orbital forcing frequency.

Effects of Viscous Layers on Rotation. Using a single layer, completely elastic ice shell allows for a calculation of k and a prediction of a single libration amplitude [3]. We found, however, that introducing a lower ice layer with a viscoelastic or fluid rheology increases the value of k. An increase in k results in a reduction in λ, the elastic libration amplitude. If we consider a multi-layer ice shell model with layers of varying viscoelasticity we can define an "effective elastic thickness" defined by an equivalent two-layer model with a completely elastic upper shell and a viscous lower shell that contributes inertia, but not elastic restoring. It should be noted that there are many layering models which can result in the same elastic thickness due to combinations of layering rheology. The elastic upper portion of ice is the only portion which contributes to elastic restoring and therefore the value of k. The lower ice does not contribute to elastic restoring, but must be considered in determining the moment of inertia of the shell, C.

We use the method of Goldreich and Mitchell (2010) [3] to calculate elastic libration amplitudes for ice shells having varying effective elastic thicknesses. Fig. 2 demonstrates that it is the ratio of elastic thickness to total thickness which determines the libration amplitude. Observations of Galilean satellite rotation rate variations can therefore be used in conjunction with this elastic restoring model to determine an effective elastic thickness and consequentially constrain a set of layered rheology models for icy satellites.

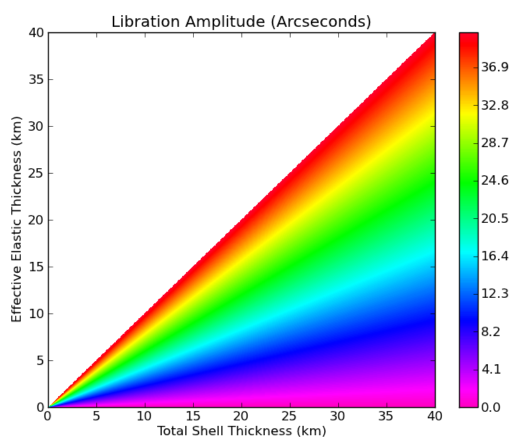

Figure 2: Europa's elastic libration amplitude for various elastic thicknesses.

Multi-layer shell models: To date, we have restricted our study to shell models having one or two layers of varying viscoelasticity. Having understood the qualitative effects that the rheology of the lower ice has on the rotation rate of the shell, our next step is to develop an analytic theory for multi-layer ice shells. We currently use a module called SatStress [4] to calculate stresses from reorientation with which we calculate elastic energy and k, and then infer the elastic libration amplitude.

This process produces clear predictions of how varying viscosities in multiple layers affect the elastic libration. If observations of rotation rate variation on Europa showed a libration amplitude of $25''$, for example, this model would predict a ratio of elastic thickness to total shell thickness of about 0.6. This ratio could be achieved through many combinations of ice shell layering. One such combination could be an ice shell with a total thickness of 40 km having an elastic thickness of 24 km, while another might be a total thickness of 25 km and elastic thickness of 15 km. Each of these elastic thickness values would then correspond to a set of models having various layer thickness and viscosities. Much of our future work concerning this idea will be dedicated towards determining analytically how ice rheology and layering influence the value of k, and therefore elastic libration amplitude.

Conclusions: Using the new concept of an elastic restoring torque [3], we derive an expression for a periodic rotation rate variation due to elastic libration of the ice shells on icy moons. We examine this process in a simple two-layer viscoelastic shell. We find that the presence of viscous lower ice layer reduces the amplitude of the rotation rate variation. Furthermore, we define a quantity called an "Effective Elastic Thickness" which summarizes a multi-layered viscoelastic model into a two layer model in which only the upper, perfectly elastic ice contributes to elastic restoring. We show that it is simply the ratio of elastic thickness to total thickness which determines elastic libration amplitude. Observations of rotation rate variation on icy moons can constrain the elastic thickness ratio and, with sufficient precision, constrain ice shell rheology and structure models.

References: [1] T. van Hoolst, et al. (2008) *Icarus* 195:386 doi. [2] H. M. Jara-Orué, et al. (2011) *Icarus* 215:417 doi. [3] P. M. Goldreich, et al. (2010) *Icarus* 209:631 doi.arXiv:0910.0032. [4] Z. A. Selvans (2009) *Time, tides and tectonics on icy satellites* Ph.D. thesis University of Colorado at Boulder.