

EVOLUTION OF ORBITS AT THE 2:3 RESONANCE WITH NEPTUNE. S. I. Ipatov (Institute of Applied Mathematics, Miusskaya sq. 4, Moscow 125047, Russia; ipatov@spp.keldysh.ru), J. Henrard (Department of Mathematics, Facultés Universitaires Notre-Dame de la Paix, Rempart de la Vierge, 8, Namur B-5000, Belgium; jhenrard@math.fundp.ac.be).

We investigate the evolution of orbits at the 2:3 resonance with Neptune. The six-body problem (the Sun, four giant planets, and a test body) is integrated using the symplectic "Swift integrator" of Levison and Duncan [1]. The initial conditions of the major planets are those proposed by these authors. The integration span is 20 Myr. Depending on the type of behaviour of the difference $\Delta\Omega = \Omega - \Omega_N$ between the longitudes of the ascending nodes of the body and Neptune and the argument of perihelion ω , we can consider several types of orbits: *DI*, *ID*, *II*, *LI*, et al. Here, the first letter corresponds to the behaviour of difference $\Delta\Omega$ and the second letter to the behaviour of ω ; a letter "I" corresponds to an increase of $\Delta\Omega$ or ω , a letter "D" corresponds to a decrease of these elements, "L" is for the case of libration, and "S" is for relatively small variations (not more than 360° during 20 Myr). For example, the type *IL* corresponds to the case when $\Delta\Omega$ increases and ω librates. In contrast to the type *L*, at the type *S* the variations don't look like as exact dependence of sin on time. The type of behaviour of $\Delta\Omega$ and ω can change during the time span of 20 Myr.

Variations in eccentricity e , inclination i , and semimajor axis a were relatively small for some initial data. For other initial data, they were large; some bodies left the resonance, and others were ejected into hyperbolic orbits. Even variations in initial orbital orientations and initial positions in orbits can cause large variations in dependencies of e , i , and a on time. For example, at $a_o = 39.3$ AU, $e_o = 0.15$, $i_o = 5^\circ$, we considered 14 different values of Ω_o , ω_o , and M_o (where M is the mean anomaly, and starting values for a body are designated by "o") and obtained that 8 of these bodies left the resonance. Among these 8 nonresonant orbits, there were 6 of the type *DI*, one of the type *SI*, and one of the type *LI*. The type *ID* was obtained for three resonant orbits, and for other three resonant orbits the type changed: *SI* \rightarrow *IL*, *SD* \rightarrow *SI* \rightarrow *ID* \rightarrow *IS*, and *SS* \rightarrow *SI* \rightarrow *SS* \rightarrow *SI* \rightarrow *IL*. The time spent inside the Kozai resonance (i.e., $\omega \approx \text{const}$) was equal to several million years.

At $\Omega_o = \omega_o = M_o = 60^\circ$, $e_o = 0.15$, and $i_o = 5^\circ$, we made runs for different values of a_o . We obtained the type *ID* at $39.1 \leq a_o \leq 39.3$ AU, the type *DI* at $38.5 \leq a_o \leq 38.9$ AU and $39.6 \leq a_o \leq 39.9$ AU, and changes in types (*SI* \rightarrow *DI*, *ID* \rightarrow *SD*, and *DI* \rightarrow *SD*) at a_o equaled

to 39.0, 39.4, and 39.5 AU. At $i_o = 5^\circ$ for e_o from 0 to 0.3 (with a step equaled to 0.05), the following types were obtained: *SI* \rightarrow *SD*, *LI* \rightarrow *LD*, *LD*, *ID*, *ID*, *II*, and *ID*. At $e_o = 0.15$, we had the type *ID* for $0 \leq i_o \leq 15^\circ$ and the types *IL*, *ID*, and *II* or some combination of these types for $30^\circ \leq i_o \leq 90^\circ$. For most of the runs, variations in the critical angle σ exceeded 180° . For nonresonant orbits we usually obtained the types *DI*, *II*, or *LI*. The types *DD* and *DL* were not obtained in our runs.

For orbits with a small amplitude of σ -libration, regions of i and e corresponding to the η_{18} ($\Delta\Omega \approx \text{const}$) and Kozai resonances are presented in Fig. 5 in [2]. These regions are located far from each other: $e < 0.03$ and $i < 10^\circ$ for the η_{18} resonance, and $e > 0.2$ at $i < 10^\circ$ for the Kozai resonance. In one our run at $e_o = 0.05$ and $i_o = 5^\circ$, $\Delta\Omega$ librated around 180° with an amplitude $\sim 180^\circ$ and at the same time ω librated around 270° with an amplitude $\sim 100^\circ$ during 6 Myr. For other values of Ω_o , ω_o , and M_o at $e_o = 0.05$ and $i_o = 5^\circ$, we usually obtained the type *LI*, but sometimes also the type *DI*. According to Fig. 5 in [2], ω decreases at $e < 0.2$ and $i < 10^\circ$. We obtained a lot of orbits with increasing ω at these values of e and i . A region of values of e and i , for which the η_{18} resonance was obtained, was much larger than that in this figure.

For many runs, i varies quasi-periodically with time with a period equal to several million years, and $\Delta\Omega$ changes by 360° during this period. In this case, if $\Delta\Omega$ decreases during evolution, then $\Delta\Omega = 0$ when i reaches its maximum value, and $\Delta\Omega = 180^\circ$ when i reaches its minimum value. If $\Delta\Omega$ increases, then we have 180° and 0 for maximum and minimum values of i , respectively. We often obtained variations in e and i with a period equaled to $T_\omega/2$, where T_ω is the time, during which ω decreased or increased by 360° . For all considered runs, maximum values of e and i exceeded 0.07 and 3° , respectively. Interval $\Delta a = a_{max} - a_{min}$ of variations in a for the resonance is about 1 AU.

For *SI*, this work was supported by the Russian Foundation for Basic Research, project no. 96-02-17892, and by the ESO grant no. B-06-018.

[1] Levison, H.F., and Duncan, M.J.: 1994, *Icarus*, **108**, 18-36.

[2] Morbidelli, A., Thomas, F., and Moons, M.: 1995, *Icarus*, **118**, 322-340.