MUTUAL GRAVITATIONAL INFLUENCE OF BEYOND-NEPTUNE BODIES. S. I. Ipatov, Institute of Applied Mathematics, Miusskaya sq. 4, Moscow 125047, Russia, ipatov@spp.keldysh.ru.

A lot of papers devoted to the Kuiper belt were published last years (see a review by Weissman (1995)). Ipatov (1988b, 1995) numerically investigated evolution of orbits of three gravitating beyond-Neptune bodies circling the Sun. Basing on these results and dependencies of the evolution of the average eccentricity e_{av} in the disc of bodies on the number and masses of the bodies constituting the disc, I obtained that if the disc consists of identical bodies with masses close to the masses of the discovered objects of the Kuiper belt, then e_{av} grew up to 0.1 and average inclination i_{av} grew up to 4° during the age T_{SS} of the Solar System, if the total mass of the disc equaled to several Earth masses. Below I consider variations in semimajor axis a of beyond-Neptune bodies due to their mutual gravitational influence.

Stern (1995) considered 35,000 objects with radius $r \geq$ 100 km and explored the rate of collisions among bodies in the present-day Kuiper disc. He adopted a particle in a box formalism for calculations of collision rates. The same approach was used by Fernandez (1980) for estimates of average times elapsed between close (up to a radius r_s of the considered sphere) encounters of beyond-Neptune bodies. The spheres used by Fernandez were less than the sphere of action (i.e., the Tisserand sphere), so the role of gravitational interactions was understated. The particle in a box approach is based only on the average velocity but it does not take into account various orbits of bodies. Davis and Farinella (1996) used some modifications of Öpik's formulas for calculations of the probability p_{ij} of a collision of a pair of bodies.

For the case when semimajor axes a, eccentricities e, and the angle between the orbital planes, Δi, do not change before a collision, Ipatov (1988a) obtained the following formula for the characteristic time T_c elapsed up to the collision of two bodies circling the Sun, :

$$T_c \approx 2\,\pi^2\,\xi\,\sin\Delta i\;T_s\;(R/r_\Sigma)^2/(k_\varphi\;k_\Theta)\;, \qquad (1)$$

where $k_\Theta = 1 + (v_p/v_r)^2$, $k_\varphi = \Delta\varphi/r_s^*$, $r_s^* = r_s/R$, $\Delta\varphi$ is the sum of angles (in radians) with apices in the Sun, within which the distance between the orbit of the first body and the projection of the second orbit on the plane of the first orbit is less than r_s, T_s is the synodic period of revolution, r_Σ is the sum of bodies' radii, v_p/v_r is the ratio of parabolic and relative velocities of the bodies, R is the distance of bodies from the Sun, and ξ may vary between 0.5 and 1 depending on the considered model. Appraisals presented below are made for $\xi = 1$. For $\xi = 0.5$ they are smaller by a factor of 2. In contrast to the approach, used by Öpik (1951), Arnold (1965), and other scientists, T_c depends on T_s and k_φ.

For actual beyond-Neptune objects, Δi usually varies before a collision due to variations in i, ω, and Ω caused by gravitational influence of planets. Therefore, in this case it is better to consider the model, for which Δi varies between 0 and $\Delta i_{\max}$ before encounters up to r_s and $\sin\Delta i \approx \Delta i$. For this case Ipatov (1988a) obtained the formula that uses $\Delta i_{\max}/\eta$ instead of $\sin\Delta i$ in formula (1), where $\eta = 0.5 + \ln(\Delta i_{\max}/r_s^*)$. We denote by T^* the values of T_c for such model with variable Δi. The ratio of T^*/T_c equals to $\eta/2$ at $\sin\Delta i \approx \Delta i = \Delta i_{\max}/2$. At $\Delta i_{\max} = 10°$ we have $\eta/2 \approx 4.4$ for ratios of masses of bodies to the mass of the Sun $\mu_1 = 5\cdot10^{-12}$ close to those for the observed beyond-Neptune bodies and $\eta/2 \approx 6$ for bodies, which masses are smaller by a factor of 1000.

Typical values of T^* are obtained to be equal to $3.5\cdot10^{16}$ yr for $r_\Sigma = 200$ km, $R = 40$ AU, i varing from 0 to 10°, and $T_s = 2P$, where P is the period of a revolution of a body around the Sun. The value of the average intrinsic collision probability, which corresponds to the above value of T^*, equals to $7\cdot10^{-22} km^{-2} yr^{-1}$ and is of the same order as that obtained by Davis and Farinella (1996). If we suppose that a typical beyond-Neptune object crosses orbits of $2\cdot10^4$ objects, then the time elapsed up to its collision with one of such objects is $\sim 2\cdot10^{12}$ yr. So less than 1% of these objects could collide with other such objects during T_{SS}. If we suppose that the number of bodies with radius $r > r_*$ is k^2 times greater than the number of bodies with radius $r > r_* k$, then we obtain that a body with radius $r = 100$ km collides with some body with radius $r \geq 10$ km in $7\cdot10^8$ yr.

The probability and deterministic methods (Ipatov, 1992, 1993b) may be used for a choice of the pairs of bodies encounting up to the radius of the considered sphere. For the probability method, the pair of encounting bodies is chosen proportionally to the probability p_{ij} of their encounter. For the deterministic method, the time τ_{ij} (where $\tau_{ij} \propto 1/p_{ij}$) elapsed until the isolated (from other bodies) encounter of the pair of encounting bodies is minimum. Results of computer runs showed that if the number of bodies in the disc is not small (i.e., each body can cross the orbits of several other bodies), then for the deterministic algorithm the velocity of disc evolution is by a factor of ten (or more) greater than that for the probability algorithm. So in the Kuiper belt characteristic times between collisions can be several times smaller than those presented above and obtained for the probability approach.

A characteristic time between a close encounter of two identical objects up to the radius r_{st} of the Tisserand sphere is about $4\cdot10^{10}$ yr for $a = 40$ AU, $\mu_1 = 5\cdot10^{-12}$, and Δi varying from 0 to 10°. If the orbit of the object can cross the orbits of $2\cdot10^4$ objects of the same size, than for a probability approach it takes part in $N_{en} \approx 2\cdot10^3$ encounters up to r_{st} during T_{SS}. For the deterministic approach, N_{en} may exceed $2\cdot10^4$. To my opinion, an actual evolution of the disc consisted of a large number of objects may be closer to the deterministic approach.

For $\mu_1 = 5\cdot10^{-12}$ at $e = 0.05$ and $\Delta i = 0$, the values of da/a are about 10^{-4}, where da is the mean value of the difference between variations in a during one encounter up to r_{st} (Ipatov, 1995). For a spatial model, the values of da/a may be smaller by a factor of 2 or 5. If the body is located in the middle of the belt of almost identical bodies, then da_Σ, its mean variations in a during N_{en} encounters, may be $\sim da\cdot N_{en}^{1/2}$.

In this case, even for $N_{en} \approx 2 \cdot 10^4$ at $a = 40$ AU and $da/a = 10^{-4}$, we have $da_\Sigma < 0.1$ AU. The difference of semimajor axes of two gravitating bodies have a tendency to increase (until it will become about $a \cdot e$). So if the body is at the edge of the disc, then da_Σ may be closer to $da \cdot N_{en}$ than to $da \cdot N_{en}^{1/2}$.

Ipatov (1995) obtained that $da \propto \mu_1^{1/2}$ at $e \geq 50(\mu_1)^{1/3}$. Therefore, variations in a for bodies with masses $\mu_1 \sim 5 \cdot 10^{-12}$ during $N_{en} = 2 \cdot 10^4$ encounters up to r_{st} will be of the same order than those for bodies with masses $\mu_1 = \mu_{pl}$ at $N_{en} = 10^3$, where μ_{pl} is the ratio of the mass of Pluto to the mass of the Sun. Our runs of evolution of three and four identical gravitating bodies with $\mu_1 = \mu_{pl}$ showed that variations in a for some bodies exceeded 10% during 10^3 encounters. So variations in a of some actual beyond-Neptune bodies during T_{SS} could exceed 1 AU.

Our results of investigations of evolution of orbits of gravitating objects - material points with $\mu_1 \sim 10^{-9} - 10^{-8}$ at $e \approx 0.05 - 0.2$ and $\Delta i = 0$ showed that for every $N_{en}^* \sim 10^3 - 10^4$ encounters up to r_{st} there is such encounter, for which variations in a are about $a \cdot e$. The difference between the values of a of two encounting objects can change a sign at such encounters. In particular, two bodies of identical masses can exchange their values of a. Basing on results of the runs with $\mu_1 \sim 10^{-9} - 10^{-5}$, I evaluated that $N_{en}^* \sim 10^4 - 10^5$ at $\mu_1 \sim 5 \cdot 10^{-12}$. The above jumps in a were obtained for the planar model. Let us suppose that for a spatial model the ratio n_r of the number of encounters up to r_{st} to the number of such jumps is of the same order as that for the planar model. In this case, for a deterministic approach during T_{SS} a body may have on the average one chance to change considerably its semimajor axis (for a probability approach this chance is smaller by an order of magnitude). For the Kuiper belt, most of these very close encounters may not finish by collisions of objects, and the objects can change their semimajor axes considerably. Probably, n_r is smaller for a spatial model, but the results presented above indicate that, due to gravitational influence of the largest bodies of the Kuiper belt, variations in a for some bodies may exceed several AU during T_{SS}, and separate bodies can migrate from the middle and external parts of this belt to the internal part of the belt.

Duncan *et al.* (1995) showed that there is a region (36 AU $\leq a \leq$ 39 AU) with $i \leq 15^\circ$ and $e \leq 0.05$, which is dynamically stable for the age of the Solar System. The observations to date indicate that this region is unpopulated. Our results show that some bodies could left this region due to the gravitational influence of the largest bodies of the Kuiper belt.

Our results of investigations of evolution of discs of bodies (Ipatov 1993a) obtained using the spheres method showed that the eccentricities of the largest bodies of the disc are smaller on the whole than the average eccentricity and these bodies can increase their semimajor axes due to gravitational interactions with those smaller bodies, which decrease their semimajor axes. The same conclusions may be true also for the Kuiper belt, in which some bodies migrated to the orbit of Neptune. Therefore, a fraction of large bodies (among all bodies) may be larger for the central part of the Kuiper belt than for the inner part.

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