EVOLUTION OF THE ORBITS OF THE OBJECTS P/1996 R2 (LAGERKVIST) AND P/1996 N2 (ELST-PIZARRO).
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Two new interesting objects were observed recently in the asteroid belt. The object P/1996 R2 (Lagerkvist) has a Jupiter-crossing orbit. On August 7, 1996 Eric W. Elst reported his discovery of the object with a tail, which looked like a comet (Marsden, 1996a). This object was named P/1996 N2 (Elst-Pizarro), has a typical asteroid orbit, and was identified with the object 1979 OW7.

The model of calculations

Two series of calculation were made for the orbits close to that of the object **P/1996 R2 (Lagerkvist)**. In each series, the calculations were made for 12 objects into the future and into the past. In the first series, orbital elements of the basic object were taken to be equal to $a = 3.7905223$ AU, $e = 0.3126849$, $i = 2.60367^\circ$, $\Omega = 40.27673^\circ$, and $\omega = 334.41016^\circ$ and it was considered that the object passes its perihelion on 1997 Jan. 20.38224 TT. Here, a is the semimajor axis, e is the eccentricity, i is the inclination, Ω is the longitude of ascending node, and ω is the argument of perihelion. Such orbit was obtained by Nakano basing on 80 observations 1996 Sept. 11 - Oct. 9. For the second series, the basic orbital elements were taken to be equal to $a = 3.7863247$ AU, $e = 0.3111012$, $i = 2.60485^\circ$, $\Omega = 40.25683^\circ$, and $\omega = 334.18825^\circ$ and it was considered that the object passes its perihelion on 1997 Jan. 19.70954 TT. This orbit was obtained by basing on 90 observations 1996 Sept. 11 - Oct. 16 (Ref. MPC 28052). Besides the basic orbit, in the first series we considered ten orbits, for which orbital elements varied by ± 0.02 AU in a, ± 0.008 in e, $\pm 0.006^\circ$ in i, $\pm 0.1^\circ$ in Ω, and $\pm 1^\circ$ in ω. For the second series, such variations about another basic orbit were equal to ± 0.004 AU in a, ± 0.0016 in e, $\pm 0.0012^\circ$ in i, $\pm 0.02^\circ$ in Ω, and $\pm 0.23^\circ$ in ω. The last variations are close to the difference between orbital elements of two basic orbits. For the twelfth case in each series, we considered the basic orbit from another series, but the date of perihelion passage was identical for all runs in a series.

Time steps for plots equaled to 100 yr for the BULSTO integrator, i.e, for the method by Bulirsh and Stoer (1966), and to 1000 and 500 yr for the program RMVS3 (regularized mixed variable symplectic) from the SWIFT package by Levison and Duncan (1994) for the first and second series of runs, respectively. Using the RMVS3 integrator, we considered the evolution of objects until their ejection into hyperbolic orbits. For the runs with the use of the BULSTO integrator, the time span equaled to $T = \pm 2$ Myr.

Orbital elements of the object **P/1996 N2 (Elst-Pizarro)** were taken to be equal to $a = 3.15619$ AU, $e = 0.16749$, $i = 1.38424^\circ$, $\Omega = 160.266^\circ$, and $\omega = 133.295^\circ$ at JDT 2450200.5 (i.e., Epoch 1996 Apr. 27.0) (Marsden, 1996c). At that time the mean anomaly $M \approx 1.52^\circ$. Below we shall call this orbit as a basic orbit. Orbital elements of ten test objects with orbits close to the basic orbit were also considered. Their orbital elements differed from the above values by ± 0.005 AU for a (first two objects), by ± 0.015 for e (second pair of objects), by $\pm 0.05^\circ$ for i (third pair), and by $\pm 2^\circ$ for Ω and ω (last four objects). Only one orbital element was varied for each object. Other orbital elements and the initial value of M were the same as those for the basic orbit. For a, e, and i, these differences are about 5 times greater than the differences between two sets of these elements presented by Marsden (1996b,c). The values published by Marsden (1996b) are from 15 observations July 14 - Aug. 21 and those presented in (Marsden, 1996c) are based on 29 observations from 21 of July 1979 until 21 of August 1996.

The orbital evolution of P/1996 N2 and close orbits was investigated by the BULSTO integrator for a time span $T = \pm 1$ Myr (into the future and into the past). The orbital elements were determined with a time $\Delta t = 200$ yr for $T = \pm 1$ Myr and $\Delta t = 10$ yr for $T = \pm 0.1$ Myr. Influence of all $n_{pl} = 9$ planets was taken into account. For $T = \pm 20$ Myr we used the RMVS3 integrator and took $\Delta t = 1000$ yr. For the basic orbit, the integration was also made during $T = 200$ Myr in the future. All planets, exclusive Mercury, ($n_{pl} = 8$) were taken into account for the runs with the use of the RMVS3 integrator. Using the BULSTO integrator, we showed that for $T = 0.1$ Myr the values of intervals $\Delta e = e_{max} - e_{min}$ and $\Delta i = i_{max} - i_{min}$ obtained for $n_{pl} = 9$ and $n_{pl} = 8$ are differed only by 0.000015 and 0.00004°, respectively, where 'max' and 'min' indicate maximum and minimum values of orbital elements during the considered time span. For the considered time step of integration equaled to 30 days for the RMVS3 integrator, the accuracy of integrations (in comparison with the results obtained with the use of the BULSTO integrator) was better for $n_{pl} = 8$. For example, at $T = 0.1$ Myr the difference between the values of Δe obtained by using the RMVS3 and BULSTO integrators was ~ 0.002 for $n_{pl} = 9$ and ~ 0.0005 for $n_{pl} = 8$. For the differences between the values of Δi, we have 0.0014° and 0.0003°, respectively.

Object P/1996 R2 (Lagerkwist)

The orbit of the object P/1996 R2 (Lagerkvist) was found to be very chaotic. In the considered runs a lifetime T_e (before ejection into a hyperbolic orbit) of such objects varied from 0.05 to 27 Myr for integration into the future and from -0.03 to -9.26 Myr for integration into the past. Using the BULSTO integrator we obtained the median lifetime to be equal to -0.4 and 0.6 Myr and to -0.8 and 0.2 Myr for the first and second series of runs, respectively. For the RMVS3 integrator, these values were about -0.45 and 0.3, and -0.25 and 0.5 Myr, respectively. Among 48 runs obtained with the use of BULSTO integrator, the absolute values of T_e were less than 0.1 Myr for 6 cases, exceeded 2 Myr for 3 runs, and were between 1 and 2 Myr for 8 cases. Among these 48 runs, there were 10 runs, for which the minimum perihelion distance $q_{min} < 1$ AU. In one run $q_{min} = 0.002$ AU. Usually objects spend part of their lifetime in resonances with the planets. While integrating into the past, during some time interval we obtained one object in a low eccentric orbit in the beyond-Neptune belt.

In 1990 (at JDT 2448061.45) there was the close encounter (up to 0.21 AU) of the object P/1996 R2 with Jupiter that caused large variations in orbital elements of the object. Duration of these valuable variations was about 4 yr. The next close encounter with Jupiter (up to 0.57 AU) will be in 2052 (at JDT 2470864.6). Only in 1996 the minimum distance of the object P/1996 R2 from the Earth became less than 2 AU.

Object P/1996 N2 (Elst-Pizarro)

All orbits, which were close to that of the object P/1996 N2 (Elst-Pizarro), were found to be stable at the considered time interval (up to 200 Myr). They were located outside the 2:1 resonance with Jupiter. For the object P/1996 N2 (Elst-Pizarro), it was obtained that $\Delta a = a_{max} - a_{min} \approx 0.041$ AU, $\Delta e \approx 0.11$, and $\Delta i \approx 3.5^\circ$. For 10 close objects, variations in Δa and Δe do not exceed 11% (from the values of Δa and Δe, respectively) for variations in $a_\circ$ equaled to ± 0.005 AU, and variations in Δa are about ± 7% for variations in $e_\circ$ equaled to ± 0.015. Here "$\circ$" denotes initial values. For all other considered variations in initial data, variations in Δa, Δe, and Δi do not exceed 2%.

In all considered runs the values of ω increased by 360° during a time span $T_\omega \approx 6000$ yr, the values of $\Delta\Omega = \Omega - \Omega_J$ decreased by 360° during a time span $T_\Omega \approx 2 \times 10^4$ yr, and the values of $\Delta\tilde{\omega} = \tilde{\omega} - \tilde{\omega}_J$ increased by 360° during a time span $T_{\tilde{\omega}} \sim 10^4$ yr, where $\tilde{\omega} = \Omega + \omega$, Ω_J and $\tilde{\omega}_J$ are the values of Ω and $\tilde{\omega}$ for Jupiter.

For the object P/1996 N 2 (Elst-Pizarro), a period of main variations in a was obtained to be about $T_a \approx 110$ yr. The amplitude of variations in e with this period was about 0.02-0.03. For such variations, the minimum of e corresponds to the maximum of a and the maximum of e corresponds to the minimum of a. A period of main variations in e, $T_e \approx 8.4 \cdot 10^3$ yr and i varies with a period $T_i \approx 1.2 \cdot 10^4$ yr, which is a little greater than T_e. There are also variations in e and i with periods $T_{eJ} \approx 0.054$ Myr and $T_{iJ} \approx 0.049$ Myr, respectively, where T_{eJ} and T_{iJ} are periods of variations in e and i for Jupiter (and the same are those for Saturn). The amplitude of variations in e with a period T_e varies from 0.06 to 0.1 during T_{eJ}: it is maximum when eccentricity e_J of Jupiter is minimum and it reaches minimum when e_J is maximum. All these variations are caused by Saturn. Variations in e with a period T_e are identical during evolution if we consider the three-body problem: the Sun-Jupiter-asteroid.

The object P/1996 N2 (Elst-Pizarro) has the smallest inclination among the actual asteroids with close values of a, and so it has the largest probability of a collision with other asteroids. As for most of other nonresonant asteroids (Ipatov, 1992), $\Delta\tilde{\omega}$ for the object P/1996 N2 (Elst-Pizarro) increases by 360° during T_e and $\Delta\tilde{\omega} \approx 0$ when e reaches its maximum and $\Delta\tilde{\omega} \approx 180^\circ$ when e reaches its minimum.

Values of i are smaller when inclination i_J of Jupiter is minimum. During $T_{iJ} \approx 4T_i$ one or two minimum values of i are close to 0. At these moments the values of $\Delta\Omega$ and ω jumps by 180°. During other time, $\Delta\Omega$ varies about 0 with a period T_i and an amplitude about 90°.

While varying initial orbital elements, we obtained variations in $T_{\tilde{\omega}}$ to be about ∓ 4% and ∓ 3% for variations in $a_\circ$ and $e_\circ$ equaled to ± 0.005 AU and ± 0.015, respectively. In these cases variations in T_i are equal to about ∓ 2% and ∓ 4%, respectively. For other variations in intial data, variations in $T_{\tilde{\omega}}$ and T_i are much smaller.

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