LUNAR ORBIT EVOLUTION AND TIDAL HEATING OF THE MOON:

W.M. Kaula and C.F. Yoder, Department of Geophysics & Space Physics, Univ. of California, Los Angeles, 90024

Introduction. There have been several recent suggestions that tidal dissipation may have been a significant heat source in the moon (e.g., 1, 2), as well as recurrent hypotheses that resonance locks of the moon's orbit (1, 3, 4) have delayed the outward evolution of the moon's orbit, thus keeping the moon close enough for tidal heating to be appreciable.

Our previous examinations of these problems led to negative conclusions (5, 6, 7, 8). By and large, our conclusions stand. However, there may have been appreciable enhancement of tidal amplitudes in the moon due to (a) excursions of the rotation axis with the evolution of Cassini's law (9) or (b) inhomogeneities of lunar properties; the analysis (7) of resonance with earth harmonics was incomplete; and possible couplings of external bodies to the perigee motion, rather than the longitude motion, of the moon were overlooked in (8).

Dissipation. For a moon modelled as an elastic body with only radial variations in properties, negligible inclination of the equator to the orbit, and imperfections in elasticity represented by the dissipation factor $1/Q$, the principal dissipation is given by (6, eq. 19): $\dot{E}_{2201} = -\frac{1}{2}\left[D^*_{2201}\right]^2\left(\dot{v}^*_{2201} - 2\dot{\theta}\right)E_{22}/Q$

where D^*_{2201} is the factor dependent on semi-major axis and eccentricity to order e, v^*_{2201} is the orbital angle $2(\omega+\Omega)+3M$, θ is the sidereal angle of the moon's rotation, and E_{22} is the unit coefficient strain energy per unit volume, of rather complicated form. The present rate of dissipation is about $10^{18}/Q$ ergs/sec (6, eq. 24). Given the high seismic Q in most of the moon, (10^4 in the outer 500 km), and the lower limit on the order of 1/3 of the ratio of tidal Q to seismic Q now inferred for the solid mantle of the earth, it seems difficult to justify more than about 3×10^{15} ergs/sec present tidal dissipation in the moon. This is negligible compared to the 10^{19} ergs/sec radiogenic energy estimated from heat flow measurements.

This dissipation rate is proportionate to $e^2/a^{15/2}$, while, if dissipation is dominant in the earth, $\dot{e}/e$ is proportionate to $\dot{a}/a$ (6, eqs. 5, 18, 45). Hence for other distances a_i, $\dot{E}_i/\dot{E}_o \approx (a_o/a_i)^{11/2}$. For example, at $a/R_\oplus = 34.2$, $\dot{E}$ was about

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0.7×10^{17} ergs/sec by this calculation. But at 34.2 $R_\oplus$ the moon went through large oscillations in obliquity, due to passage from one Cassini state to another (9). Hence there is an amplification by about $(\sin I/e)^2$ giving a maximum rate of 9×10^{19} ergs/sec. This condition was transitory, and within 25 $R_\oplus$ the $(a_o/a_i)^{11/2}$ rule applies again, so that the orbit must be taken back to about 14 $R_\oplus$ for present radiogenic heating to be equaled.

Tidal dissipation varies nearly inversely with mean rigidity, so long as $19\mu/2\rho g R \geq 1$: i.e., down to about .05 the present rigidity (cf. 10). The main effect of radial inhomogeneity in μ, as suggested by the volcanism $3.2 - 4.0 \times 10^9$ years ago, is to increase strain energy significantly in the layers above and below a zone of low μ. Given a decrease in Q as well, this could be a mechanism of broadening the zone of approach to melting over that obtained by thermal history calculations <u>if</u> the moon was within, say, 25 $R_\oplus$. Numerical models of this hypothesis are being calculated.

<u>Resonance.</u> The tidal equations for mean motion and eccentricity (6) are of the form $\dot{n}_T(e) = -Ka^{-8}(13 - 7\alpha)$, $\dot{e}_T = e\dot{n}_T(o)(5 - 7\alpha)/3$, where the factor α is the ratio of the moon's kR^5/M^2Q to the earth's: about 0.01 at present.

Resonance by coupling with the earth's gravity field places a minimum on the coefficient $C_{\ell m}$ of the field. The most plausible resonance is with C_{22} and has an angle $\varphi = 3\lambda - \tilde{\omega} - 2\theta$, where θ is the earth's sidereal angle. This resonance occurred at $a = 3.2\ R_\oplus$ (corresponding to a 5^h day). Given an established resonance, $\dot{n} = \dot{n}_{RES} + \dot{n}_T \approx 0$ and $\dot{e} \approx \dot{e}_{RES} \approx -2\dot{n}_{RES}/9ne$. Since $\dot{n}_T < 0$, $\dot{e} < 0$. But $\dot{n}_{RES} \propto eC_{22}\sin\varphi$, implying that the libration amplitude of φ must increase, leading to dissipation of the resonance.

More promising are resonances involving the Jovian and solar 'evections', i.e. commensurabilities with the lunar perigee motion, for which the libration angle $\varphi = 2(\tilde{\omega} - \lambda_p)$, in which λ_p is the Jovian or solar longitude. From $\dot{\tilde{\omega}} \approx 3J_2(R/a)^2 n/2 + 3n_\oplus^2/4n$, $\dot{\tilde{\omega}}_s = \dot{\lambda}_s$ at $a = 53.4\ R_\oplus$ and $\dot{\tilde{\omega}}_s = \dot{\lambda}_\oplus$ at $4.3\ R_\oplus < a < 6.3\ R_\oplus$. The gravitational coupling is $\sim(\partial\dot{\tilde{\omega}}/\partial e^2)(m_p/m_\odot)n^2e^2\sin\varphi$; hence the resonance lock is stable, because it implies e must

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increase. Capture occurs automatically if the initial eccentricity $e < 0.0076$ for Jovian resonance and $e \leq 0.1$ for solar.

If $\alpha \geq 2$, $\dot{n}_T$ tends to zero as e increases to a critical value. However, for the Jovian case it is necessary to have unreasonably high tidal friction in the moon, $\alpha = 3.5$, to damp the eccentricity to the present value, .055. In the much earlier solar resonance, it is plausible to assume a fluid earth and frequency-dependent Q, whence $2 < \alpha < 6$. Furthermore, the $\dot{\varphi}$ term in the second order equation (11) feeds energy back into the libration if $\alpha \geq 2$. The lifetimes of these resonances for $\alpha = 3.5$ are $\Delta T_J \approx .25 \ln |.06/e_0| \times 10^9$ yr and $\Delta T_\odot \approx 10^3 \ln |.2/e_0|$ yr.

Another effect of the Jovian resonance is to decrease the inclination: from 17° to 7°, if it lasts 10^9 years. The solar resonance has negligible effect on the time scale of evolution, but would lead to great tidal heating and the transfer of enough angular momentum out of the earth-moon system to account for the excess required by fission hypotheses. It does not affect the inclination.

Conclusions. (1) the increase in dissipation due to the Cassini transition (9) was of slight effect because it was too far out, 34.2 $R_\oplus$; (2) an appreciable increase in dissipation (maybe by 20) occurred if there was a thick "soft" zone; (3) the moon had to have been within 25 $R_\oplus$ in any case for significant dissipation; (4) the earth's gravity & solar resonances are at distances too close to reconcile with plausible accretion origin (12); (5) the earth's gravity resonance is unstable; (6) the long term stabilization of evective resonances results in growth of lunar eccentricity requiring a later damping difficult to reconcile with present thermal state.

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