
A new computerized Markov Chain technique has been used to simulate the statistics of large lunar craters. With this model we examined the density at which crater saturation occurs, accurately modeled the observed lunar highlands' crater size-frequency curve from 7 to 1270 km diameter, and derived an explanation of the highly degraded appearance of the lunar highlands. Where appropriate, these simulations included the effects of crater and basin ejecta as specified by McGetchen et al. (1).

Woronow (2) reported the results of a Monte Carlo simulation of crater saturation and equilibrium where computer limitations restricted the study to a diameter range of 16x. The Markov Chain technique easily manipulates dynamic ranges of more than 180x. To gain this advantage one sacrifices information on regional variations in the crater density and retains information on only the average crater density. Each crater is represented by 8 points on its rim; subsequent impacts degrade craters by removing 0 to 8 of those points. For each \( \sqrt{2} \) diameter interval a vector describes the fraction of craters in each degradation state. In addition, a scalar describes the number of craters in each diameter interval. The program utilizes 21 different transformation matrices to detail the effects of each new crater on all pre-existing craters as a function of the new crater's size, the old crater's size and degradation state, and the size of the counting surface. The Markov Chain computer program follows this sequence of events: (1) a new crater of a random size is generated from a specified distribution; (2) all state vectors are multiplied by the appropriate transformation matrix to produce the new state vectors; (3) the fresh crater is factored into the appropriate state vector and scalar.

The first application of this model examined the results of the earlier Monte Carlo saturation study by Woronow (2). That study indicated that even the most densely cratered lunar highlands had large-crater densities well below the saturation limit, but the simulation suffered from a short dynamic range which could not simultaneously account for large basin-forming events and smaller scale cratering. Because of the amount of computer time necessary to bring a large dynamic range to saturation, the Markov simulation was stopped short of saturation but at a density well above the "densely cratered lunar highlands" of Strom and Whitaker (3). At equivalent "model times" the Monte Carlo and the Markov Chain results agree excellently (Figure 1); therefore, this independently verifies the results of the Monte Carlo study and the conclusion remains that even the most densely cratered lunar highlands are not at or near either the saturation or equilibrium density limits.

Two attempts to simulate the observed "average lunar highlands" crater curve of Strom and Whitaker utilized two different production functions. Because a -2 slope log-log function (N-D-2) has long been regarded as the progenitor of the highlands craters, this production function was tried first but failed to match the observed curve. Figure 2 illustrates how the production function is imprinted on the surface and eventually surpasses the observed densities with only the slightest trend toward conforming to the observed curve. To force this model to mimic the observed density at 7 km...
diameter, 30 fresh basins would now have to form - this would cause a severe mismatch for the rest of the curve and an over-abundance of fresh basins. Because the observed crater densities lie far below the saturation limit, one could assume that they retain the essential characteristics of the production function. The second production function tried consisted of many straight log-log segments joined together to closely emulate the observed highlands crater curve. Figure 3 shows that this production function reproduces the gross features of the average highlands crater curve at the correct crater densities. This particular run produced a slight excess of basins. When this excess is combined with the fact that the model liberally assumes that the basins' excavation diameters equal their outermost ring's diameters, the slight mismatch of the lefthand portion of the graph is not surprising. The correct production function must closely resemble the observed curve.

Finally, a re-examination of the Monte Carlo results yields an explanation of how the highlands retain a production-function crater population yet appear highly battered and degenerated. The relative number of craters produced at any two diameters is obviously dependent upon the production function used. The Monte Carlo simulations demonstrate that the relative number of craters obliterated at any two diameters is also dependent upon the production function used. The interaction of these two relationships explains the appearance of the highlands. Figure 4 illustrates the production and obliteration relationships for various production slopes and for two crater diameters differing by a factor of 4 (an arbitrarily selected ratio). A -3.0 slope, for instance, loses 250 craters of size D for every one it loses of size 4D, while producing only 64 of size D for every one of size 4D. This causes a population of -3 slope to rapidly change toward a -2 slope. A -1.2 slope (approximately that observed over the diameter range 7 to 56 km), however, loses craters in nearly the same proportions as they are produced. The production function's size-frequency distribution, in this case, is not readily altered even by substantial losses of craters. Simulations also show that when the total density of craters in the diameter range 7 to 56 km match the observed crater densities, a -3.0 production function has sustained minimal loss of craters while a -1.2 production function lost nearly 30% of all craters formed. Therefore, if the highlands craters resulted from a steep production function they should look quite fresh, but the shallow -1.2 production function will yield a population quite battered in appearance - yet retaining the features of the production function.

These studies indicate that the large-crater population of the lunar highlands will appear quite degraded (as it does), but still have a crater density well below the saturation or equilibrium limits and retaining the essential features of its production function.

FIGURE CAPTIONS:
1. Comparison of the Monte Carlo and Markov Chain saturation results (production slope = -2) and the observed lunar curve. P is the fraction of surface area in each 2 diameter interval (i.e., number of craters in the interval times their mean area divided by the area of the counting surface).
2. An attempt to simulate the observed lunar curve with a -2 slope production function. Successive curves are labeled in thousands of craters produced. This model fails to match the observed curve.
3. An attempt to simulate the observed lunar curve with a production function resembling the observed distribution. This model fits well.
4. Relative production and obliteration rates for two crater diameters a factor of 4 different and for various production slopes.