

ON A PROBLEM OF EVOLUTION OF ORBITS OF PHOBOS AND DEIMOS. A. A. Nakhodneva¹ and N. I. Perov², ^{1,2}Astronomical observatory, YSPU, Respublikanskaya, 108. 150000, Yaroslavl, Russia. E-mail: perov@yspu.yar.ru

Introduction: As it is known modern theories for investigation of motion of planetary satellites for the cosmogonical intervals of time are very complicated [1]. Here in order to estimate the variations of the secondary satellites orbital elements due their evolution the simple formulae are presented.

The Particular Case of Twice Averaged Model Hill Problem with Allowance for the Oblateness of the Central Body: We consider in accordance with [2-4] the model of the motion of Phobos and Deimos around an oblique Mars with the allowance for the potential of attraction of the Sun, that moves in a circular orbit of radius a_1 in a reference frame connected with the center of mass of Mars. Let us refer the angular elements of the satellites to the plane of motion of the perturbing body and to fixed direction in space (toward the point of vernal equinox) and denote by i , Ω , and ω the inclination, longitude of ascending node, and the argument of periarion, respectively, and by a and e the semimajor axis and eccentricity of the orbit of the satellites, respectively. We then average the perturbing function over mean anomalies of the perturbing body (the Sun) and the satellite to obtain (in the absence of some specific conditions) to obtain in the Hill approximation ($a/a_1 \ll 1$) quadrature (1)

$$2 \int_{w_{\min}}^{w_{\max}} \frac{dw}{\sqrt{\theta}} = P \quad (1)$$

For the determination of the allowed values of the parameters of the satellite orbit. Here, P is the characteristic period of the motion of the pericenter of the orbit of the satellite (we did not come across quadrature (1) (which, it appears, is impossible to invert [2, 3])),

$$\theta = [4\gamma c_1 w^7 - 4/3\gamma w^5 + w^2(-c_2 + 2 + 2c_1) - 4] \times [-4\gamma c_1 w^7 + 4/3\gamma w^5 - 10c_1 w^4 + w^2(8 + 8c_1 + c_2) - 6], \quad (2)$$

$$w = (1 - e^2)^{-1/2},$$

c_1, c_2, γ are the constants of the given model problem.

With the help of the equation (3)

$$\theta = 0 \quad (3)$$

we may estimate the region of variations of e , and the formula for c_1

$$c_1 = (1 - e^2) \cos^2 i \quad (4)$$

gives the regions of variations of i , where

$$c_2 = 2\gamma(1 - e^2)^{-3/2}(1/3 + \cos 2i) + 2(e^2 - \sin^2 i) + e^2 \sin^2 i(5 \cos 2\omega - 3), \quad (5)$$

$$\gamma = 1/2\alpha / \beta, \alpha = -3/8I_2(a_0/a)^2, \beta = 3/16\mu_1 / \mu(a/a_1)^3,$$

μ and μ_1 are the products of the gravitational constants by the masses of the central (Mars) and perturbing bodies (the Sun), respectively; I_2 is the coefficient at the second zonal harmonic of the gravitational field of the central body (Mars); a_0 its mean equatorial radius.

Examples: In the frame of this twice averaged restricted three body problem (the Sun-Mars-a Martian satellite) with allowance for the oblateness of Mars simple calculations in accordance with formulae (3) and (4) give

Phobos

$$e_{\min} = 0.015099997884, \quad i_{\max} = 1.0750000975 \text{ deg};$$

$$e_{\max} = 0.015100000692, \quad i_{\min} = 1.0749999681 \text{ deg}.$$

Deimos

$$e_{\min} = 0.0001999987047, \quad i_{\max} = 1.793000000474 \text{ deg};$$

$$e_{\max} = 0.0002000092704, \quad i_{\min} = 1.792999996606 \text{ deg}.$$

For these examples datum from [5] and [6] are used.

Conclusion: It is obviously the equations (1), (3), (4), (5) may be used for estimating of variations of parameters of orbits of natural and artificial satellites of Mars (as well as for estimating of the variations of parameters of orbits of the satellites of other planets, especially in cases when solar gravitating perturbations are compared with perturbations from the obliquity of the planet).

It should be noted in one of the special cases of the twice averaged Hill problem with the allowance for the oblateness of the central body (which in the general case is not integrable), when the equatorial plane of the central body coincides with the orbital plane of the perturbing body, we obtained for the first time a quadrature (1) for the determination of the period of the variation of the argument of the pericenter of satellite orbit.

References: [1] Burns J. et al. (1977) *Planetary Satellites*. [2] Lidov M.L. and Yarskaya M.V. (1974) *Kosm. Issled.*, 12, 2, 155-170. [3] Vashkovyakov M.A. (1996) *PaZh*, 22, 12, 231-240. [4] Perov N.I. (2003) *SSR*, 37, 2, 183-192. [5] http://ssd.jpl.nasa.gov/sat_elem [6] <http://nssdc.gfc.nasa.gov/planetary/factsheet/marsfact.html>