INFERRING SPATIAL PATTERNS OF ACCUMULATION FROM RADAR INTERNAL LAYERS. M.R. Koutnik\(^1\), E.D. Waddington\(^1\), and D.P. Winebrenner\(^2\). \(^1\)University of Washington, Dept. of Earth and Space Sciences, Box 351310, Seattle, WA 98195, mkoutnik@u.washington.edu, edw@ess.washington.edu. \(^2\) University of Washington, Applied Physics Laboratory and Dept. of Earth and Space Sciences, Box 355640, Seattle, WA 98195, dpw@apl.washington.edu

**Introduction:** The spatial pattern of accumulation over an ice cap is a fundamental parameter when inferring the ice-flow and climate histories. Internal layers, taken to be isochrones, contain information about accumulation patterns in both space and time. The large body of radar data from terrestrial ice caps has greatly increased our understanding of ice-sheet evolution and climate.

Deeper, older layers reflect conditions further in the past, but they have been more affected by horizontal gradients in strain rate and accumulation. The depth of a deep internal layer does not directly reflect the accumulation rate at that point. Therefore, this information is highly valuable but more difficult to interpret. Formally solving this inverse problem is necessary to determine the correct accumulation pattern recorded by deep internal layers.

Internal layers have recently been observed on the Martian ice caps by the MARSIS ice-penetrating radar [2], and we expect that more layers will be observed with SHARAD. Assuming there was a time when ice-flow controlled the shape of the Martian ice caps, we can recover some of the information which remains in the deep internal layers. Waddington et al. [1] solved this inverse problem on terrestrial ice sheets. Here, we apply the method to Martian ice.

**Method:** A steady-state flow-band model is used to calculate layer evolution [1]. Layer prediction in this forward calculation depends on the layer age, and on the accumulation-rate pattern. Inverse methods are used to find physically reasonable values of these unknown parameters. The preferred parameters generate an internal layer that fits the data within a defined tolerance.

**Forward Model.** Figure 1 shows a typical flow band. The forward model uses the governing equations for ice flow along with boundary conditions, and parameter values, to calculate the steady-state ice-surface topography and flow field everywhere in the flow band. Particle paths are found by integrating the velocity field. By following a number of particles over time, we can map out an internal layer of a particular age.

**Inverse Model.** The inverse model iteratively adjusts the unknown parameter values to find a solution subject to the following constraints. Since the radar data contain errors, we do not want our inversion to fit these data exactly; we do not want to force the solution to simply minimize the mismatch between the data and the forward-model prediction. In addition, we expect the accumulation rate to vary smoothly along the flow band. Using a gradient method to solve this inverse problem [3, 4], these two independent constraints can be introduced to stabilize the solution process and to ensure that the solution is physically meaningful. This can be achieved by minimizing a performance index, \(I_p\):

\[
I_p = \|m\|^2 + \nu (\|d\|^2 - T^2) \tag{1}
\]

The first term on the right is the model norm, the squared second spatial derivative of the accumulation rate integrated over the flow band. Penalizing large values of \(\|m\|^2\) prevents the solution from exhibiting roughness not required by the data. In the second term, \(\|d\|^2\) is the data norm, the sum of squared mismatches between the observations and the predictions,
normalized by the standard deviations of the data. The factor $\nu$ is a Lagrange multiplier, which is adjusted until the data norm equals a defined tolerance ($T$) that depends on the number of data. This value of $\nu$ sets the most appropriate trade-off between smoothness and fit.

**Results for Antarctica:** This method has been applied to a radar layer shown as the dotted black line in Figure 2, from Taylor Mouth, in Victoria Land, Antarctica [1]. The steady-state assumption is valid for layers at this depth; deeper layers could be used but would require a transient forward model.

![Figure 3. Accumulation-rate solution with error bars (a), inferred from the traced internal layer, shown in bold (b). Particle paths producing the modeled internal layer are shown as dotted lines (adapted from [1]). Vertical line marks the ice core.](image)

The accumulation-rate solution inferred from this internal layer is shown in Figure 3a. Spatial variations greater than the measured value at the core site were found. This variability has an important influence on the calculated depth-age scale at the core site shown by the red particle paths in Figure 3b.

**Application to the Martian North Polar Cap:** In anticipation of future radar data along presumed Martian flow lines, we have generated synthetic internal layers that would result from different assumptions about ice-cap evolution and accumulation. Then, using our inverse method, we explore how well we can recover the accumulation patterns that created the layers. Assuming a two-stage history of NPC evolution, Winebrenner et al. [6] defined a relict pre-trough surface on Titania Lobe [5], the lobe adjacent to the main North Polar Cap (NPC). Adopting that surface geometry here, we construct flow lines by following the surface gradient. We use the forward model to generate internal-layers along these flow lines resulting from various accumulation patterns. We then attempt to recover the accumulation-rate pattern that generated the layer, when no additional information about surface velocity or layer age is available.

In addition, we can explore the ability of this model to recover values of other unknown parameters. For Antarctica, we also solved for layer age. For Mars, we augment the parameter set with heat flux into the basal ice, and the ice-flow enhancement factor.

**Initial Results for the NPC:** Ice flow is assumed to be driven by surface-parallel shear stresses and the bed is assumed to be flat. Figure 4 shows internal layers generated by two prescribed accumulation-ablation regimes: (1) one zone of uniform accumulation and one zone of uniform ablation (result from [6]); (2) alternating zones of accumulation and ablation, known as accublation [7]. Other regimes will also be tested. These different internal structures retain some information about the accumulation-rate patterns that created them. Figure 4 shows internal layers for the two trial accumulation patterns. Regime (1) has an accumulation rate of 10 mm/yr and an ablation rate of 0.2 mm/yr. Regime (2) has six different accumulation and ablation zones, based on present-day trough locations. The accumulation rate is 1 mm/yr and the ablation rate is 0.2 mm/yr.

![Figure 4. Internal layers generated for two different accumulation patterns. Layers in left panel result from one accumulation zone and one ablation zone, separated by a vertical line. Layer age increases with depth from 50 Kyr to 0.5 Myr. Layers in right panel result from accublation [7]. Layer age increases with depth from 50 kyr to 5 Myr. Ages in terrestrial years.](image)

Synthetic internal layers, like those shown in Figure 4, can be used as “data” in our inverse procedure to explore the degree to which we can recover the original accumulation pattern, in preparation for analysis of actual radar data when they become available.

**References:**